DEMAND-RESPONSIVE MICROTRANSIT: DESIGN AND OPERATIONS

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Limitations of mobility landscape

**The Wall Street Journal**
January 8, 2023
Public Transit Goes Off the Rails With Fewer Riders, Dwindling Cash, Rising Crime

**Curbed**
June 4, 2021
Why Your Uber Ride Is Suddenly Costing a Fortune

**The New York Times**
October 11, 2019
Who’s Afraid of a Transit Desert?

**The Washington Post**
August 6, 2019
Uber and Lyft concede they play role in traffic congestion

Jacquillat—Demand-responsive microtransit
New opportunity: microtransit

High-capacity vehicles
Advance planning

Digital platform
On-demand operations

“Shared transportation system(s) that can offer fixed routes and schedules, as well as flexible routes and on-demand scheduling” (DoT)
Background and motivation

**Traveling Salesman Problem**
\[ \min_{\sigma} \sum_{i=1}^{n} \| x_{\sigma(i+1)} - x_{\sigma(i)} \| \]

**Traveling Repairman Problem**
\[ \min_{\sigma} \sum_{i=1}^{n} (n - i) \| x_{\sigma(i+1)} - x_{\sigma(i)} \| \]

**BHH theorem on TSP tour:**
\[ Z_{TSP} \sim \beta_{TSP} \left( \int \int_{\mathcal{K}} \sqrt{f(x)} dx \right) \sqrt{n} \]

**Theorem on TRP latency:**
\[ E(Z_{TRP}) \sim \beta_{TRP} \left( \int \int_{\mathcal{K}} g_f(x, y) dx dy \right) n \sqrt{n} \]
\[ g_f(x, y) = f(y) \left(1_{f(y) < f(x)} + \frac{1}{2} \cdot 1_{f(y) = f(x)} \right) \sqrt{f(x)} \]

→ Negative externalities in door-to-door transportation with high-capacity vehicles in large geographical areas

Blanchard, Jacquillat, Jaillet. “Probabilistic bounds on the k-TSP and the TRP”, Math. of OR, ’24

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How to avoid detours and delays?

Small-occupancy ride-pooling

Small service region

Zone-based regularization

Line-based regularization
Line-based microtransit

Reference trips

On-demand routing

Operations start

Trip request

Trip confirmed
Experimental setup: airport shuttle

- Demand from NYC taxi data, from 6 to 9 am
- Travel times from Google Maps, Uber, and OpenStreetMap
- Candidate lines from breadth-first tree search
Contributions

Microtransit Network Design model (MiND)

- **Subpath-based formulation**
  Two-stage stochastic integer optimization formulation with tight subpath-based second-stage structure

- **Double decomposition**
  Double decomposition approach: Benders decomposition, subpath-based column generation, label-setting algorithm

- **Computational scalability**
  Scalability of model and algorithm: high-quality solutions in otherwise-intractable instances

- **Practical impact: win-win outcomes**
  Significant benefits real-world experimental setup toward efficient, equitable and sustainable urban mobility
Problem statement

First-stage problem: network design and frequency planning

Second-stage problem: demand-responsive routing operations
Demand-responsive operations

Closer to the reference line

More demand-responsive adjustments
Subpath-based network

Physical network

Load-expanded network

Efficient subpath-based representation of microtransit operations: tight second-stage formulation without big-M capacity constraints
Decision variables

First-stage problem: network design and frequency planning

\[ x_{\ell t} = \begin{cases} 
1 & \text{reference trip } (\ell, t) \text{ is selected,} \\
0 & \text{otherwise.} 
\end{cases} \]

\[ z_{\ell pst} = \begin{cases} 
1 & \text{if passenger } p \text{ is assigned to trip } (\ell, t) \text{ in scenario } s, \\
0 & \text{otherwise.} 
\end{cases} \]

Second-stage problem: demand-responsive operations

\[ y_a = \begin{cases} 
1 & \text{if subpath-based arc } a \text{ is selected,} \\
0 & \text{otherwise.} 
\end{cases} \]
Two-stage stochastic optimization

Line construction costs

\[
\min \sum_{\ell \in \mathcal{L}} \sum_{t \in \mathcal{T}_\ell} \left( h_{\ell t} x_{\ell t} + \sum_{s \in S} \pi_s \sum_{a \in A_{\ell st}} g_a y_a \right)
\]

\[\text{s.t.} \sum_{\ell \in \mathcal{L}} \sum_{t' \in \mathcal{T}_\ell : t' \leq t + t_{\ell t}} x_{\ell t} \leq F, \quad \forall t \in \bigcup_{\ell \in \mathcal{L}} \mathcal{T}_\ell\]

\[\sum_{(\ell, t) \in M_p} z_{\ell ps t} \leq 1, \quad \forall p \in \mathcal{P}, \forall s \in S\]

\[\sum_{p \in \mathcal{P} : (\ell, t) \in M_p} D_{ps} z_{\ell ps t} \geq (1 - \kappa) C_{\ell t} x_{\ell t}, \quad \forall (\ell, t) \in \mathcal{L} \times \mathcal{T}_\ell, \forall s \in S\]

\[\sum_{p \in \mathcal{P} : (\ell, t) \in M_p} D_{ps} z_{\ell ps t} \leq (1 + \kappa) C_{\ell t} x_{\ell t}, \quad \forall (\ell, t) \in \mathcal{L} \times \mathcal{T}_\ell, \forall s \in S\]

\[\sum_{m : (m, n) \in A_{\ell st}} y(n, m) - \sum_{m : (m, n) \in A_{\ell st}} y(m, n) = \begin{cases} x_{\ell t} & \text{if } n = u_{\ell st} \\ -x_{\ell t} & \text{if } n = v_{\ell st} \\ 0 & \text{otherwise} \end{cases} \quad \forall \ell \in \mathcal{L}, t \in \mathcal{T}_\ell, s \in S, n \in V_{\ell st}\]

\[\sum_{a \in A_{\ell st} : p \in \mathcal{P}_{r(a)}} y_a \leq z_{\ell ps t} \quad \forall s \in S, p \in \mathcal{P}, (\ell, t) \in M_p\]

\[x, y, z \text{ binary}\]

Expected level service: coverage, walking, waiting, travel time, delay

Budget and fleet size

Passenger assignment

Target load factor

Flow balance

Consistency
Structure of subpath-based model

- **Segment-based model**
  - $O(P + CTN + C^2 LTA^2)$ variables in time-load-expanded network

- **Subpath-based model**
  - $O(CL2^A)$ variables in load-expanded network

- **Path-based model**
  - $O(2^AL)$ variables toward set partitioning formulation
### Benefits of subpath-based model

| $|\mathcal{L}|$ | Horizon | Path-based | Subpath-based | Segment-based |
|---|---|---|---|---|
| 5 | 60 | Sol. 100, CPU 117s, Arcs 3.1M | Sol. 100, CPU 19s, Arcs 34K | Sol. 100, CPU 6,633s, Arcs 30.0M |
| 5 | 120 | Sol. 100, CPU 760s, Arcs 8.6M | Sol. 100, CPU 279s, Arcs 94K | — |
| 5 | 180 | Sol. 100, CPU 801s, Arcs 9.6M | Sol. 100, CPU 345s, Arcs 130K | — |
| 10 | 60 | Sol. 101.6, CPU 1,278s, Arcs 29.1M | Sol. 100, CPU 60s, Arcs 882K | — |

**Far less variables than segment-based model: no time discretization**

**Fewer variables than path-based formulation: subpath-based decomposition quells the rate of exponential growth**
Solution algorithm

First-stage problem

Second-stage relaxation

Subpath routing problem

Benders main problem

Restricted Benders subproblem [CG main problem]

Pricing problem

Subpath variables

Benders cut

Integer Benders subproblem

Column generation

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Double decomposition structure

Benders main problem

Restricted Benders subproblem

Pricing problem

Independent scenarios and reference lines

Independent operations across checkpoint pairs
Scalability of methodology

Benefits of double decomposition algorithm in large-scale instances

Scalability of optimization methodology:
100 candidate lines, hundreds of stations, three-hour horizon
### Benefits of on-demand deviations

<table>
<thead>
<tr>
<th>Operating model</th>
<th>Average performance</th>
<th>Average level of service</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Util.</td>
<td>Dist.</td>
</tr>
<tr>
<td>Transit</td>
<td>12.24</td>
<td>15.16</td>
</tr>
<tr>
<td>Microtransit Low No</td>
<td>15.28</td>
<td>17.52</td>
</tr>
<tr>
<td>Microtransit High No</td>
<td>15.46</td>
<td>17.72</td>
</tr>
<tr>
<td>Microtransit Low Yes</td>
<td>15.90</td>
<td>18.13</td>
</tr>
<tr>
<td>Microtransit High Yes</td>
<td>16.16</td>
<td>18.57</td>
</tr>
</tbody>
</table>

**Higher passenger level of service** (less walk, shorter waits), and **higher demand coverage** (+3-4 passengers per vehicle)

**Significant performance improvements** from even limited flexibility [short deviations from reference line, all checkpoints visited]
Impact of demand density

Strongest benefits in medium-density regions, where demand consolidation is essential and fixed-route transit is not sufficient
Implications for network design

Transit network

Microtransit network

Equity and accessibility: geographic reach to under-served regions

60% broader coverage in Manhattan

3x more trip options on average
Microtransit vs. transit & ride-sharing

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<tr>
<th>Mode</th>
<th>Coverage</th>
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<th>Wait</th>
<th>Detour</th>
<th>Delay</th>
<th>Distance</th>
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<tr>
<td>Transit</td>
<td>33.6%</td>
<td>2.03</td>
<td>6.65</td>
<td>137.34%</td>
<td>-0.06</td>
<td>472</td>
</tr>
<tr>
<td>Microtransit</td>
<td>36.6%</td>
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<td>5.55</td>
<td>141.00%</td>
<td>0.03</td>
<td>468</td>
</tr>
<tr>
<td>Ride-pooling (Cap. 4)</td>
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<td>4.2</td>
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<td>13.4</td>
<td>1,883</td>
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<td>3.74</td>
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<td>Ride-sharing (Cap. 1)</td>
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<td>1.79</td>
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<td>1.79</td>
<td>5,671</td>
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Benefits of demand-responsive flexibility vs. fixed-line transit:
- less walk, shorter wait times, higher demand coverage

Demand consolidation in high-occupancy vehicles vs. ride-sharing

Benefits of adherence to reference line vs. ride-pooling:
- shorter delays at destination, with limited walk, wait and detour
Impact: environmental footprint

Smaller environmental footprint thanks to demand consolidation (vs. ride-sharing) and high demand coverage (vs. transit)
Performance assessment summary

### Efficiency

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### Equity

Reference line
- **Selected**
- **Not selected**

### Sustainability

- **Avg. Load (%)**
- **Avg. Load (absolute)**
- **Number of candidate lines**
- **Total distance (% maximum)**

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Thank you!