Demand-responsive Microtransit: Design and Operations

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MIT Mobility Initiative 09/29/2023

Limitations of mobility landscape

THE WALL STREET JOURNAL. January 8, 2023

Public Transit Goes Off the Rails With Fewer Riders, Dwindling Cash, Rising Crime

CURBED June 4, 2021

Why Your Uber Ride Is Suddenly Costing a Fortune

The New York Times October 11, 2019

Who's Afraid of a Transit Desert?

The Washington Post August 6, 2019

Uber and Lyft concede they play role in traffic congestion

New opportunity: microtransit

High-capacity vehicles
Advance planning

Digital platform
On-demand operations

"Shared transportation system(s) that can offer fixed routes and schedules, as well as flexible routes and on-demand scheduling" (DoT)

Background and motivation

Traveling Salesman Problem

$$\min_{\sigma} \sum_{i=1}^{n} \|x_{\sigma(i+1)} - x_{\sigma(i)}\|$$

Traveling Repairman Problem

$$\min_{\sigma} \sum_{i=1}^{n} (n-i) \|x_{\sigma(i+1)} - x_{\sigma(i)}\|$$

BHH theorem on TSP tour:

$$Z_{TSP} \sim \beta_{TSP} \left(\iint_{\mathcal{K}} \sqrt{f(x)} dx \right) \sqrt{n}$$

Theorem on TRP latency:

$$E(Z_{TRP}) \sim \beta_{TRP} \left(\iint_{\mathcal{K}} g_f(x,y) dx dy \right) n\sqrt{n}$$

$$g_f(x,y) = f(y) \left(\mathbf{1}_{f(y)<f(x)} + \frac{1}{2} \cdot \mathbf{1}_{f(y)=f(x)} \right) \sqrt{f(x)}$$

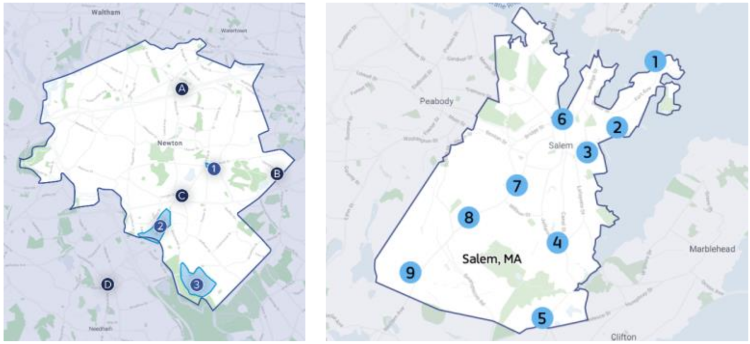
→ Negative externalities in door-to-door transportation with high-capacity vehicles in large geographical areas

Blanchard, Jacquillat, Jaillet. "Probabilistic bounds on the k-TSP and the TRP", Math. of OR, '24

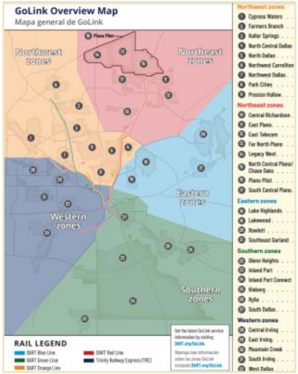
How to avoid detours and delays?

Small-occupancy ride-pooling

Small service region



Zone-based regularization



Line-based regularization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Performance and Design of Mobility Allowance Shuttle Transit Services: Bounds on the Maximum Longitudinal Velocity

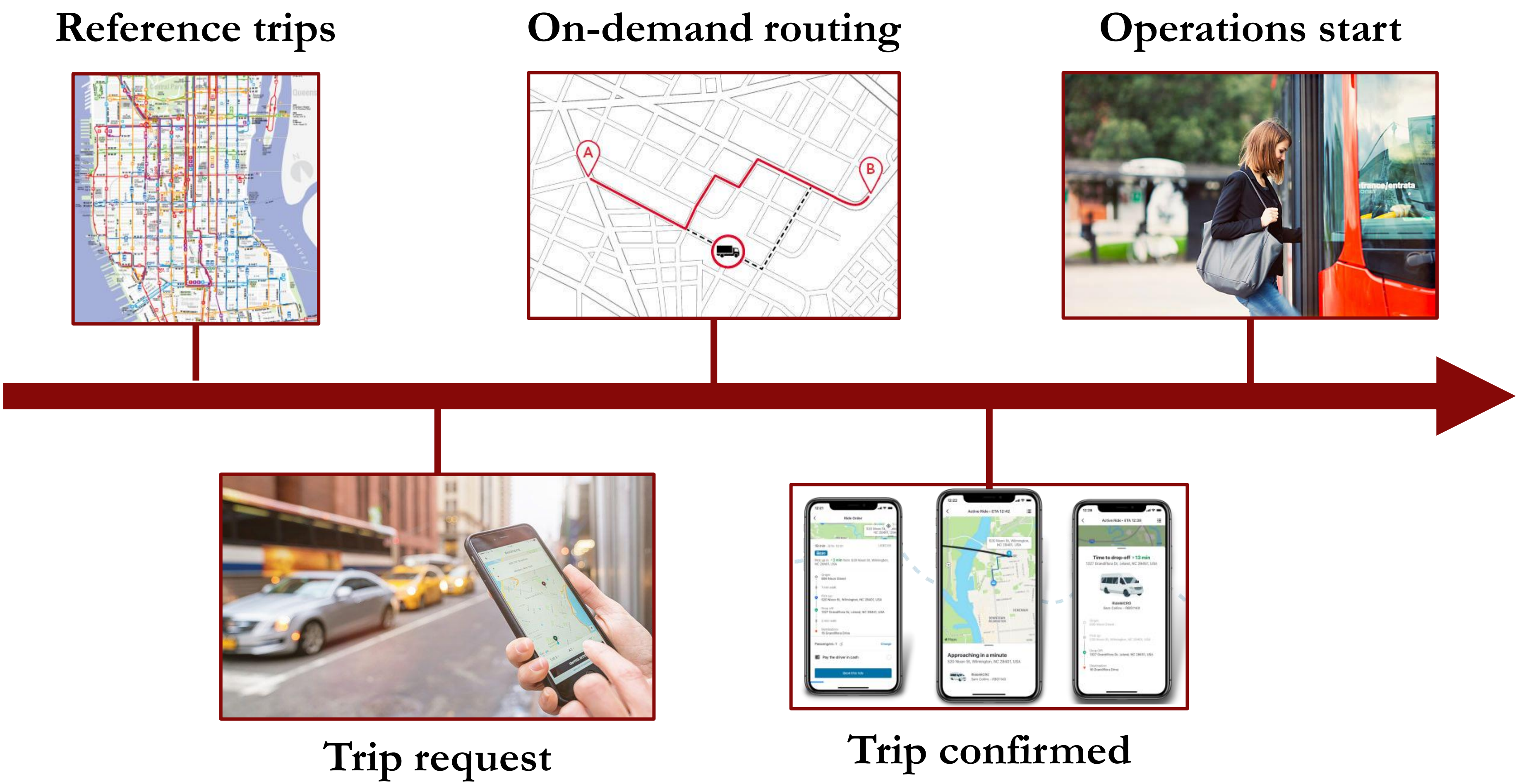
Luca Quadrifoglio, Randolph W. Hall, Maged M. Dessouky
Daniel J. Epstein Department of Industrial and Systems Engineering, University of Southern California, Los Angeles, California 90089-0193

ELSEVIER
Transportation Research Part E: Logistics and Transportation Review
Volume 149, May 2021, 102331

Improving flex-route transit services with modular autonomous vehicles

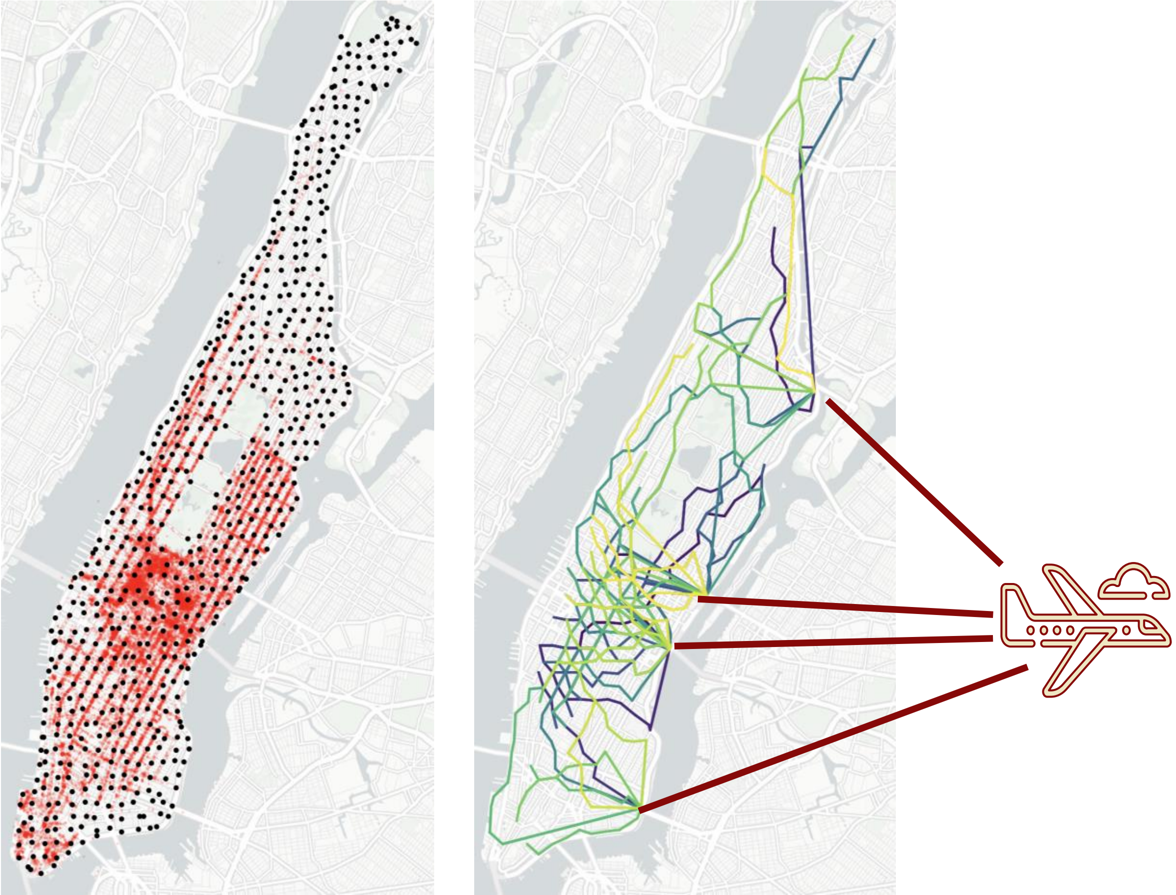
Xiaohan Liu, Xiaobo Qu, Xiaolei Ma

Line-based microtransit



Experimental setup: airport shuttle

- Demand from NYC taxi data, from 6 to 9 am
- Travel times from Google Maps, Uber, and OpenStreetMap
- Candidate lines from breadth-first tree search



Contributions

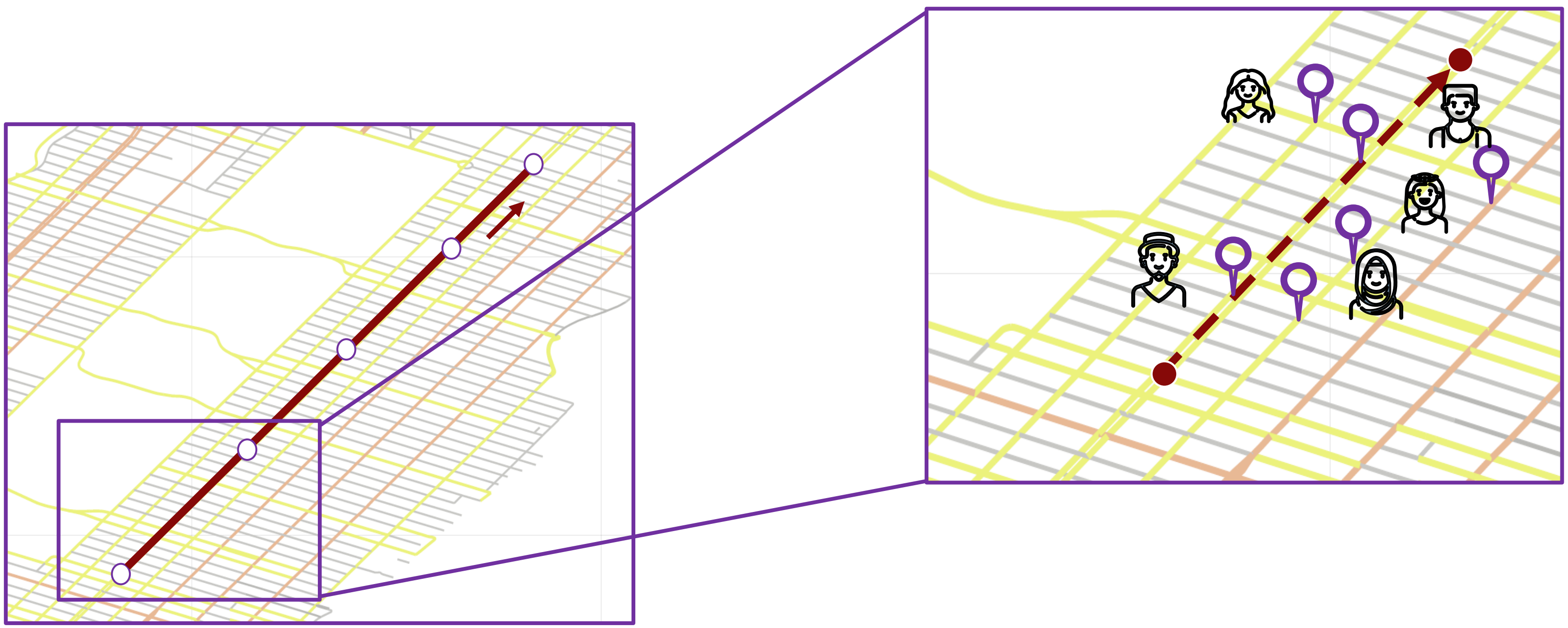
Microtransit Network Design model (MiND)

Subpath-based formulation	**Two-stage stochastic integer optimization formulation with tight subpath-based second-stage structure**
Double decomposition	**Double decomposition approach: Benders decomposition, subpath-based column generation, label-setting algorithm**
Computational scalability	**Scalability of model and algorithm: high-quality solutions in otherwise-intractable instances**
Practical impact: win-win outcomes	**Significant benefits real-world experimental setup toward efficient, equitable and sustainable urban mobility**

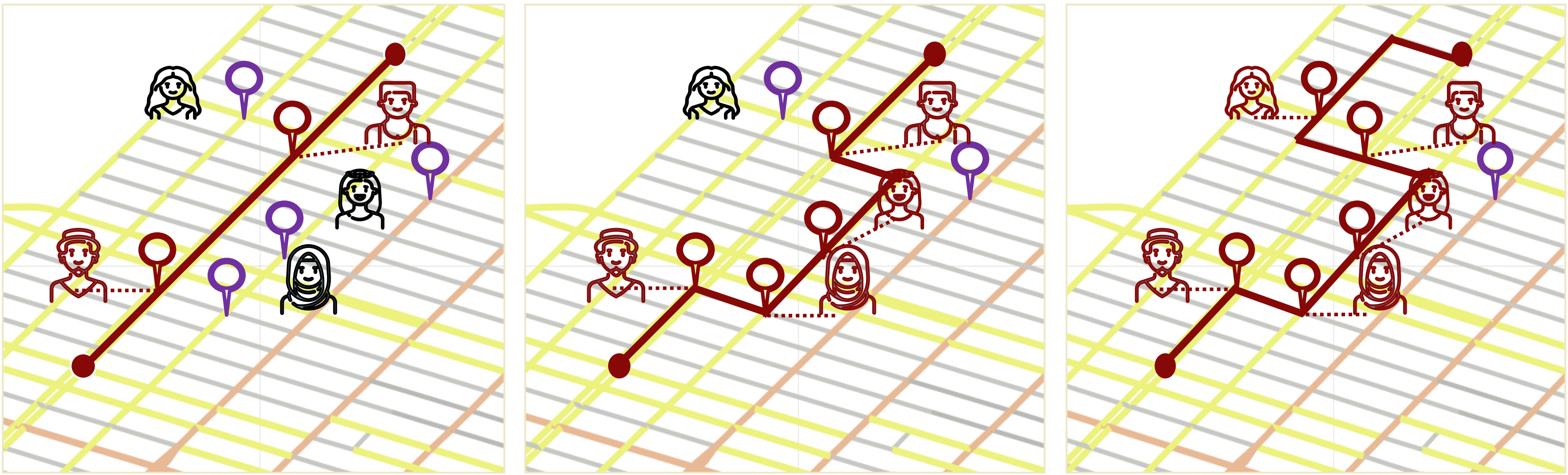
Problem statement

First-stage problem: network design and frequency planning

Second-stage problem: demand-responsive routing operations



Demand-responsive operations

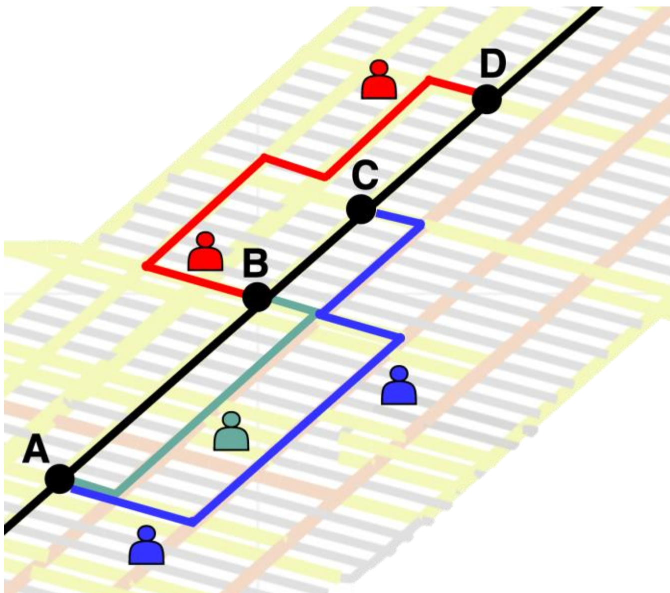


Closer to the reference line

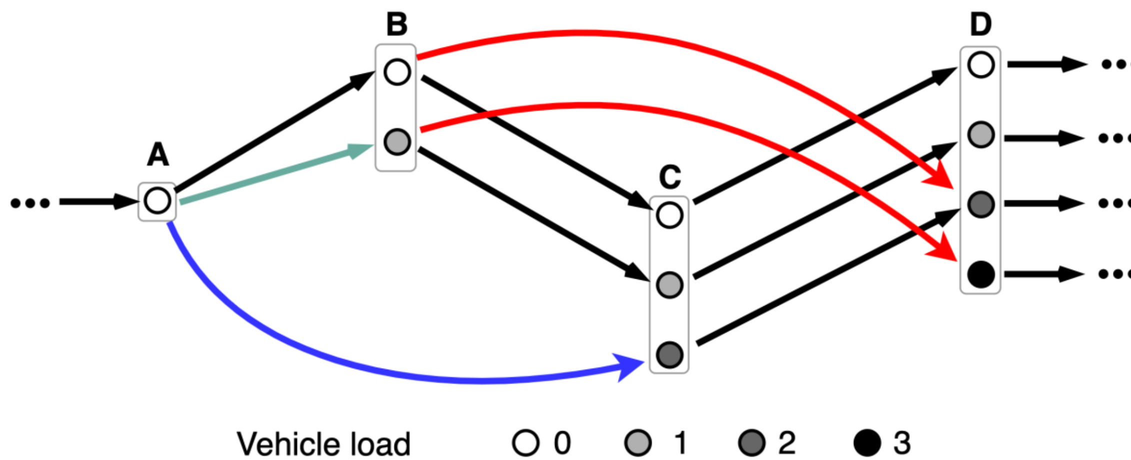
More demand-responsive adjustments

Subpath-based network

Physical network



Load-expanded network



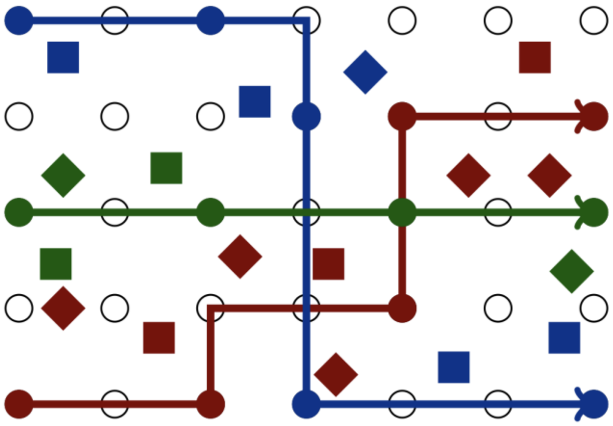
Efficient subpath-based representation of microtransit operations: tight second-stage formulation without big-M capacity constraints

Decision variables

First-stage problem:
network design and frequency planning

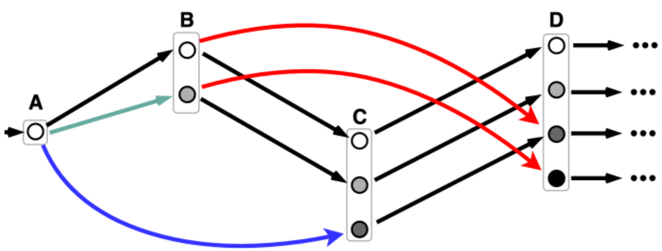
$$x_{\ell t} = \begin{cases} 1 & \text{reference trip } (\ell, t) \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{\ell pst} = \begin{cases} 1 & \text{if passenger } p \text{ is assigned to trip } (\ell, t) \text{ in scenario } s, \\ 0 & \text{otherwise.} \end{cases}$$



Second-stage problem:
demand-responsive operations

$$y_a = \begin{cases} 1 & \text{if subpath-based arc } a \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$$



Two-stage stochastic optimization

Line construction costs

Expected level service: coverage, walking, waiting, travel time, delay

$$\min \sum_{\ell \in \mathcal{L}} \sum_{t \in \mathcal{T}_\ell} \left(h_\ell x_{\ell t} + \sum_{s \in \mathcal{S}} \pi_s \sum_{a \in \mathcal{A}_{\ell s t}} g_a y_a \right)$$

$$\text{s.t.} \sum_{\ell \in \mathcal{L}} \sum_{t' \in \mathcal{T}_\ell : t' \leq t \leq t' + tt_\ell} x_{\ell t} \leq F, \qquad \forall t \in \bigcup_{\ell \in \mathcal{L}} \mathcal{T}_\ell$$

Budget and fleet size

$$\sum_{(\ell,t) \in \mathcal{M}_p} z_{\ell p s t} \leq 1, \qquad \forall p \in \mathcal{P}, \forall s \in \mathcal{S}$$

Passenger assignment

$$\sum_{p \in \mathcal{P} : (\ell,t) \in \mathcal{M}_p} D_{ps} z_{\ell p s t} \geq (1-\kappa) C_\ell x_{\ell t}, \qquad \forall (\ell,t) \in \mathcal{L} \times \mathcal{T}_\ell, \forall s \in \mathcal{S}$$

Target load factor

$$\sum_{p \in \mathcal{P} : (\ell,t) \in \mathcal{M}_p} D_{ps} z_{\ell p s t} \leq (1+\kappa) C_\ell x_{\ell t}, \qquad \forall (\ell,t) \in \mathcal{L} \times \mathcal{T}_\ell, \forall s \in \mathcal{S}$$

Flow balance

$$\sum_{m:(n,m) \in \mathcal{A}_{\ell s t}} y_{(n,m)} - \sum_{m:(m,n) \in \mathcal{A}_{\ell s t}} y_{(m,n)} = \begin{cases} x_{\ell t} & \text{if } n = u_{\ell s t} \\ -x_{\ell t} & \text{if } n = v_{\ell s t} \\ 0 & \text{otherwise} \end{cases} \quad \forall \ell \in \mathcal{L}, t \in \mathcal{T}_\ell, s \in \mathcal{S}, n \in \mathcal{V}_{\ell s t}$$

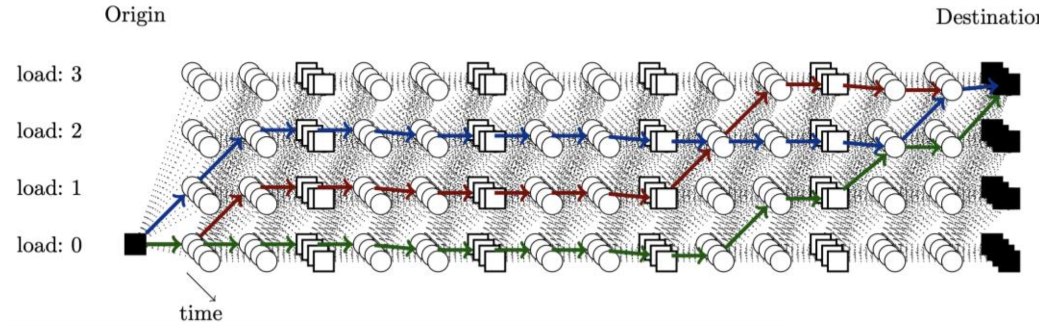
$$\sum_{a \in \mathcal{A}_{\ell s t} : p \in \mathcal{P}_{r(a)}} y_a \leq z_{\ell p s t} \quad \forall s \in \mathcal{S}, p \in \mathcal{P}, (\ell,t) \in \mathcal{M}_p$$

Consistency

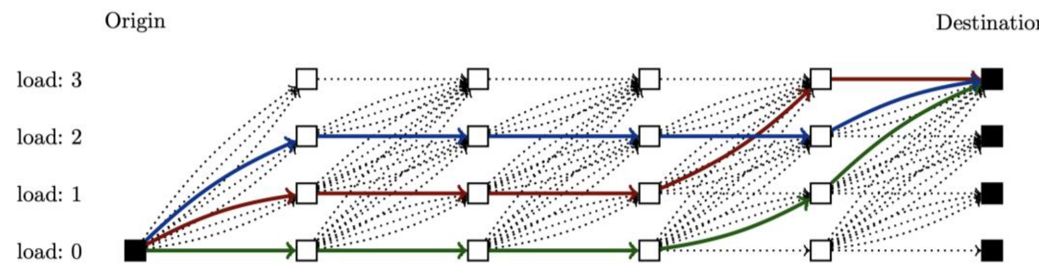
$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ binary

Structure of subpath-based model

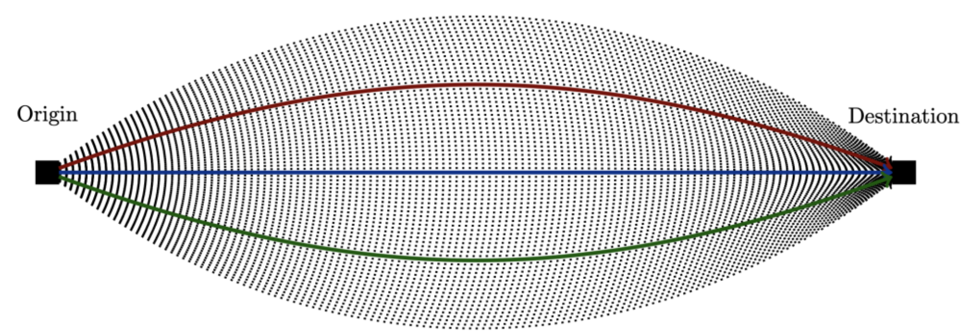
- **Segment-based model**
  - $\mathcal{O}(P + CTN + C^2LTA^2)$ variables in time-load-expanded network



- **Subpath-based model**
  - $\mathcal{O}(CL2^A)$ variables in load-expanded network



- **Path-based model**
  - $\mathcal{O}(2^{AL})$ variables toward set partitioning formulation



Benefits of subpath-based model

		Path-based			Subpath-based			Segment-based		
$\|\mathcal{L}\|$	Horizon	Sol.	CPU (s)	Arcs	Sol.	CPU (s)	Arcs	Sol.	CPU (s)	Arcs
5	60	100	117s	3.1M	100	19s	34K	100	6,633s	30.0M
5	120	100	760s	8.6M	100	279s	94K	—	—	—
5	180	100	801s	9.6M	100	345s	130K	—	—	—
10	60	101.6	1,278s	29.1M	100	60s	882K	—	—	—

Far less variables than segment-based model: no time discretization

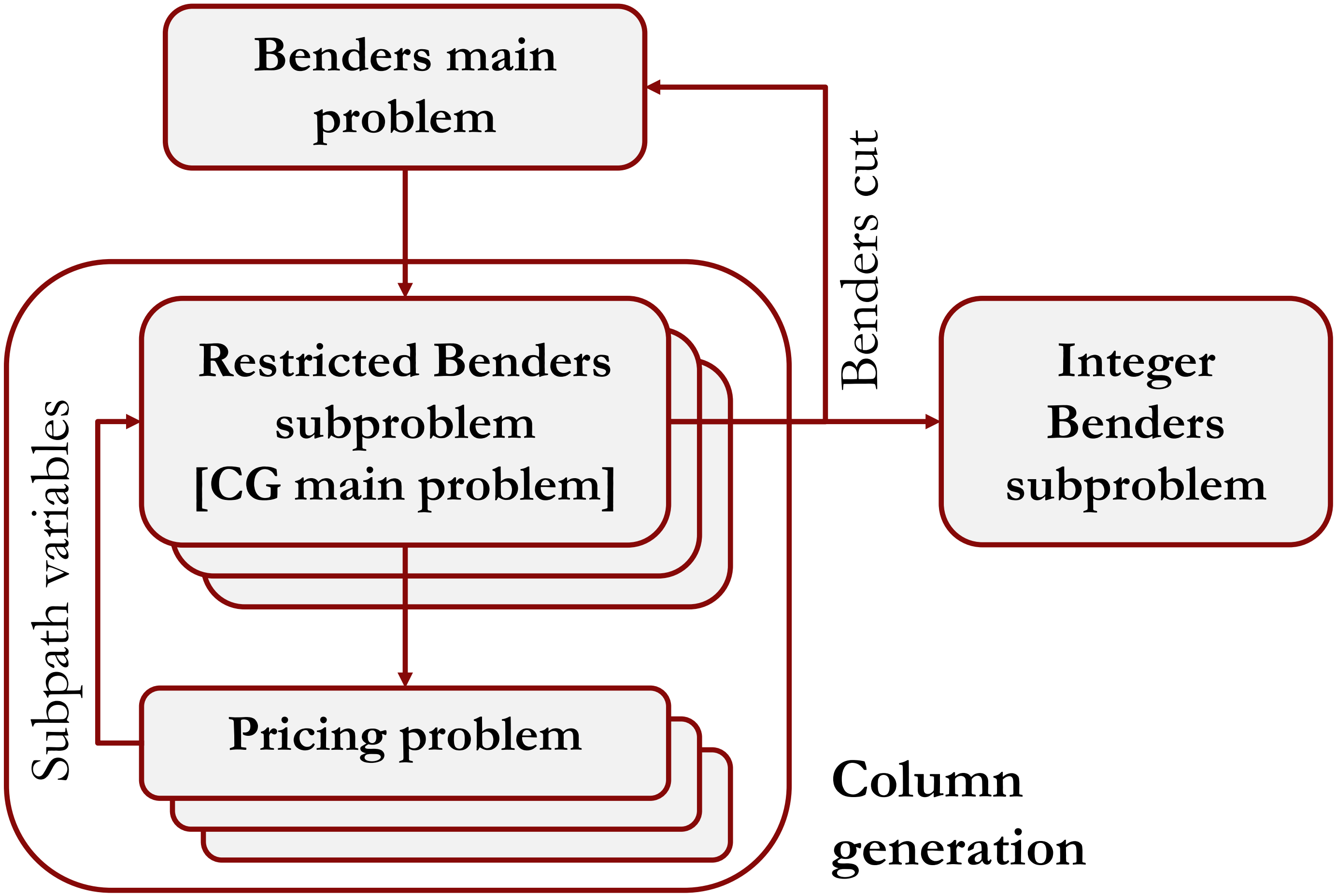
Fewer variables than path-based formulation: subpath-based decomposition quells the rate of exponential growth

Solution algorithm

First-stage problem

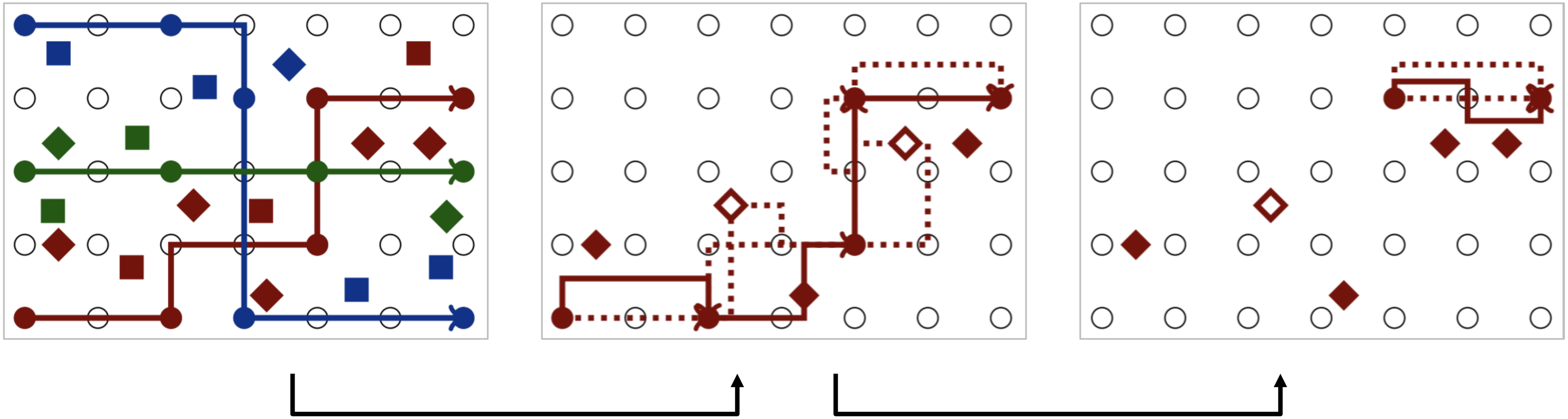
Second-stage relaxation

Subpath routing problem



Double decomposition structure

Benders main problem	Restricted Benders subproblem	Pricing problem



Independent scenarios and reference lines

Independent operations across checkpoint pairs

Scalability of methodology

		K = 0									K = 1				
		Benders			DD-E			DD-H			DD-E			DD-H	
$\|\mathcal{L}\|$	Horizon	Sol.	Gap	CPU(s)	Sol.	Gap	CPU(s)	Sol.	CPU(s)		Sol.	Gap	CPU(s)	Sol.	CPU(s)
10	60	100	**0.0**	48	100	**0.0**	82	102	57		100	**0.0**	6,222	100.3	75
	120	—	—	—	100	**0.0**	256	100.8	121		102.5	6.5	10,800	100	187
	180	—	—	—	100	**0.0**	407	101.1	200		104.4	10.7	10,800	100	280
100	60	—	—	—	100.9	1.1	10,800	100	2,802		—	—	—	100	10,800
	120	—	—	—	105.9	9.9	10,800	100	10,800		—	—	—	100	10,800
	180	—	—	—	—	—	—	100	10,800		—	—	—	100	10,800

Benefits of double decomposition algorithm in large-scale instances

Scalability of optimization methodology:
100 candidate lines, hundreds of stations, three-hour horizon

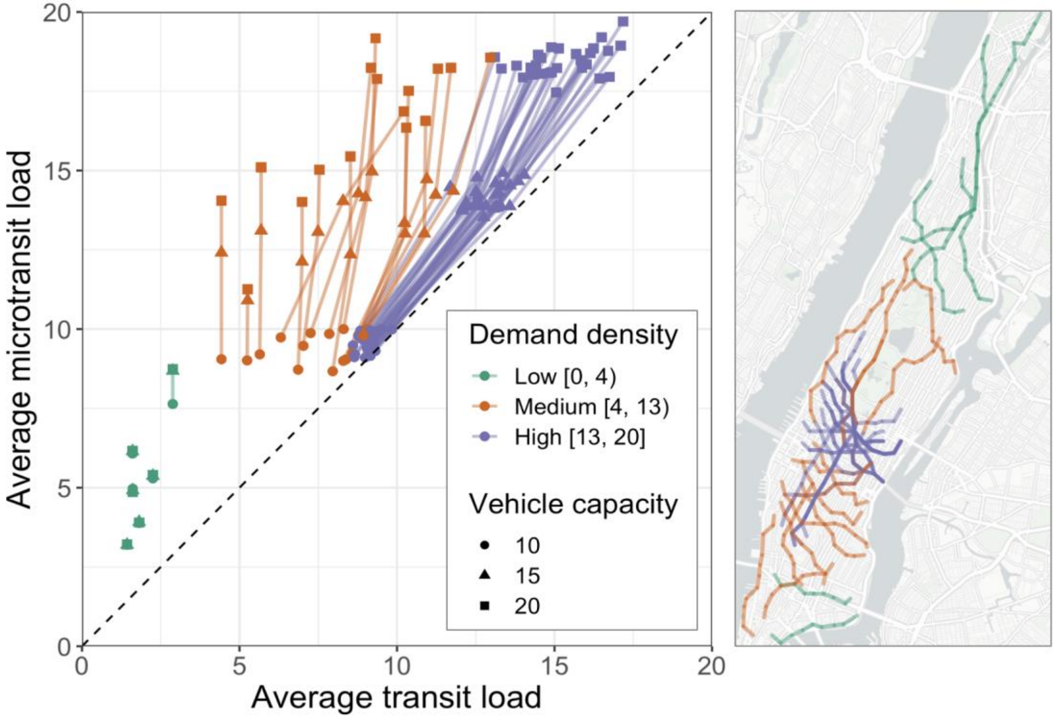
Benefits of on-demand deviations

Operating model			Average performance			Average level of service			
Mode	Dev.	Skip?	Util.	Dist.	Dist./pass.	Walk	Wait	Detour	Delay
Transit	—	—	12.24	15.16	2.34	2.30	6.94	150.38%	-1.84
Microtransit	Low	No	15.28	17.52	2.02	1.69	6.21	150.77%	-0.52
Microtransit	High	No	15.46	17.72	1.98	1.62	6.04	150.08%	-0.50
Microtransit	Low	Yes	15.90	18.13	1.96	1.48	5.25	153.51%	-0.28
Microtransit	High	Yes	16.16	18.57	1.81	1.43	5.39	151.17%	-0.33

Higher passenger level of service (less walk, shorter waits), and higher demand coverage (+3-4 passengers per vehicle)

Significant performance improvements from even limited flexibility [short deviations from reference line, all checkpoints visited]

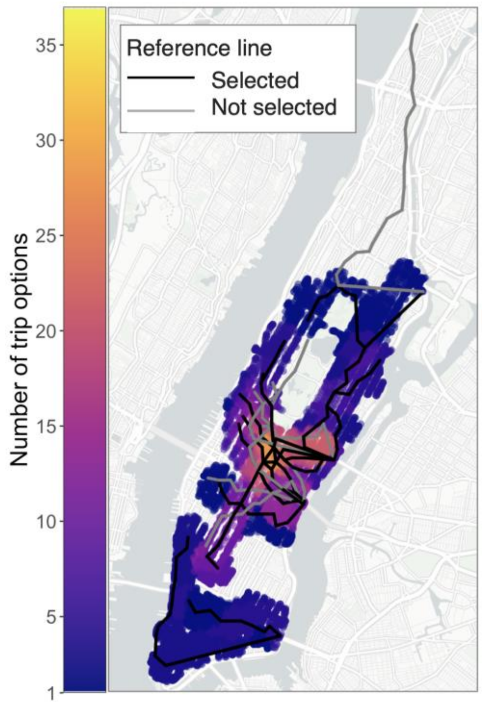
Impact of demand density



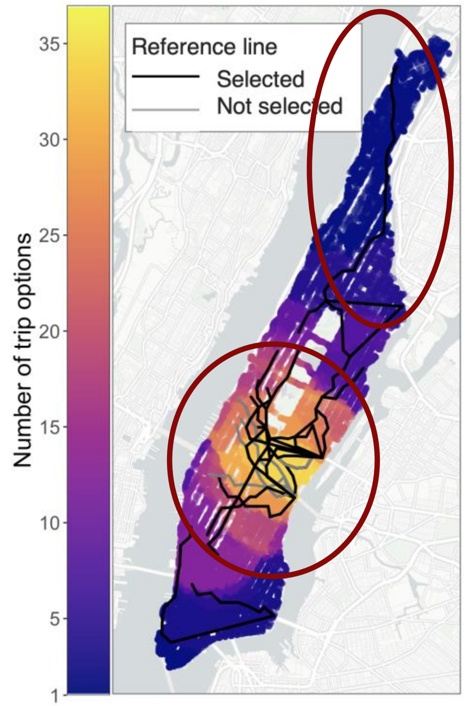
Strongest benefits in medium-density regions, where demand consolidation is essential and fixed-route transit is not sufficient

Implications for network design

Transit network



Microtransit network



60% broader coverage in Manhattan

3x more trip options on average

Equity and accessibility: geographic reach to under-served regions

Microtransit vs. transit & ride-sharing

Mode	Coverage	Walk	Wait	Detour	Delay	Distance
Transit	33.6%	2.03	6.65	137.34%	-0.06	472
Microtransit	36.6%	1.36	5.55	141.00%	0.03	468
Ride-pooling (Cap. 4)	36.3%	0	4.2	150.68%	13.4	1,883
Ride-pooling (Cap. 2)	44.7%	0	3.74	124.60%	8.17	3,359
Ride-sharing (Cap. 1)	50.5%	0	1.79	100.00%	1.79	5,671

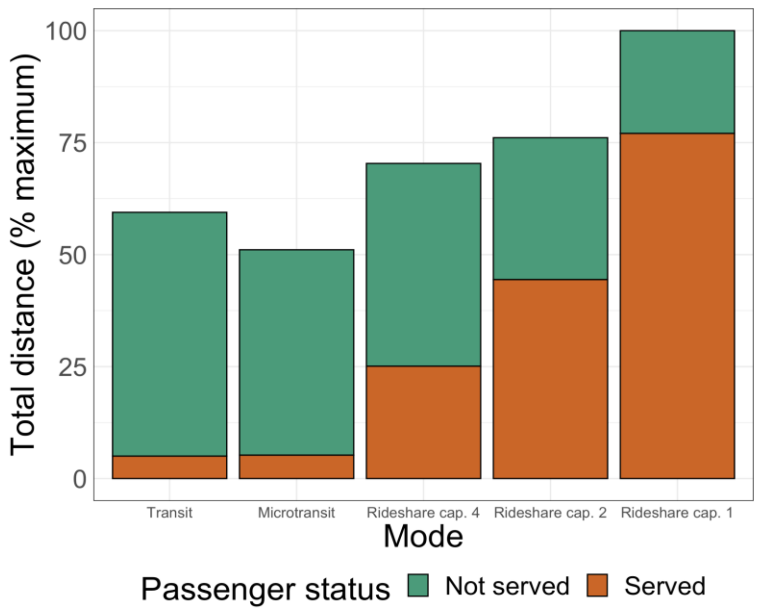
Benefits of demand-responsive flexibility vs. fixed-line transit:
less walk, shorter wait times, higher demand coverage

Demand consolidation in high-occupancy vehicles vs. ride-sharing

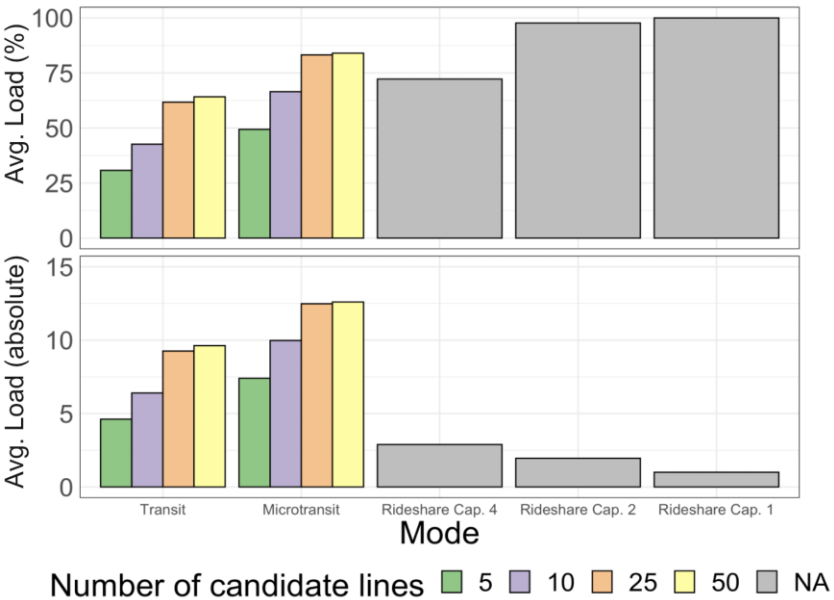
Benefits of adherence to reference line vs. ride-pooling:
shorter delays at destination, with limited walk, wait and detour

Impact: environmental footprint

Total distance traveled



Vehicle load



Smaller environmental footprint thanks to demand consolidation (vs. ride-sharing) and high demand coverage (vs. transit)

Performance assessment summary

Efficiency

Mode	Coverage	Walk	Wait	Detour	Delay	Distance
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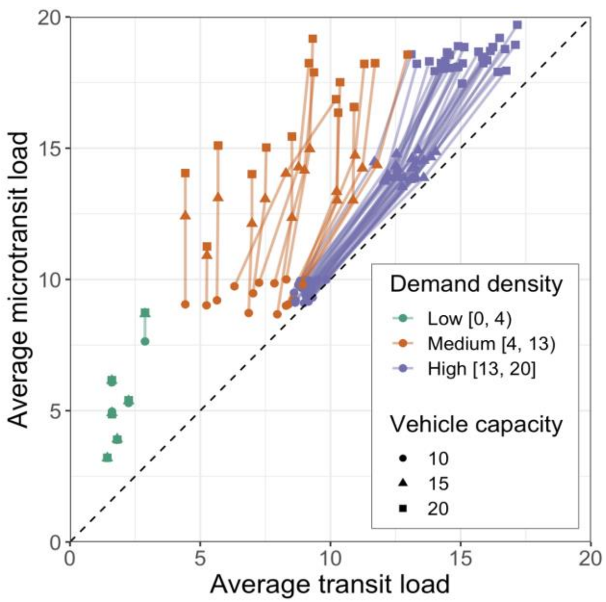
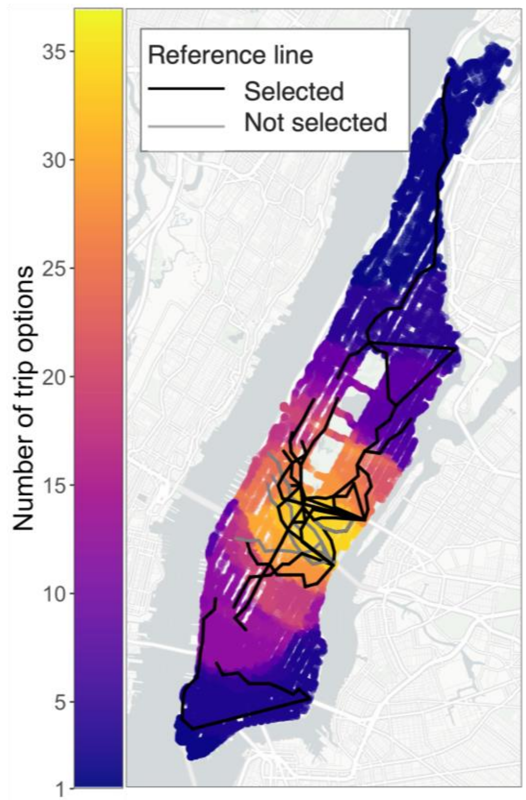
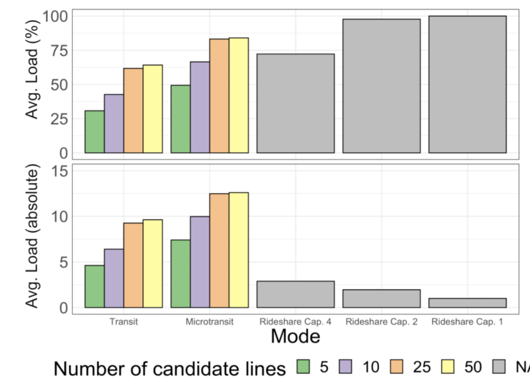
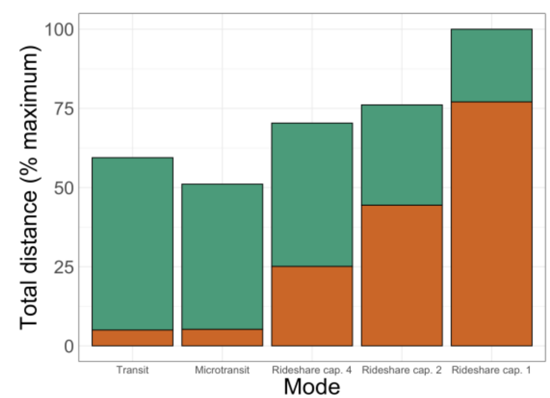


Equity



Sustainability





Thank you!