

# Co-Design of Complex Systems: From Autonomy to Future Mobility Systems

... Or ...

## Rethinking ~~AV Development~~ with ~~Foundation Models~~ Society-Critical Systems Co-Design

MIT Mobility Forum

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# Designing today's engineering systems could have positive societal impact, but is complex

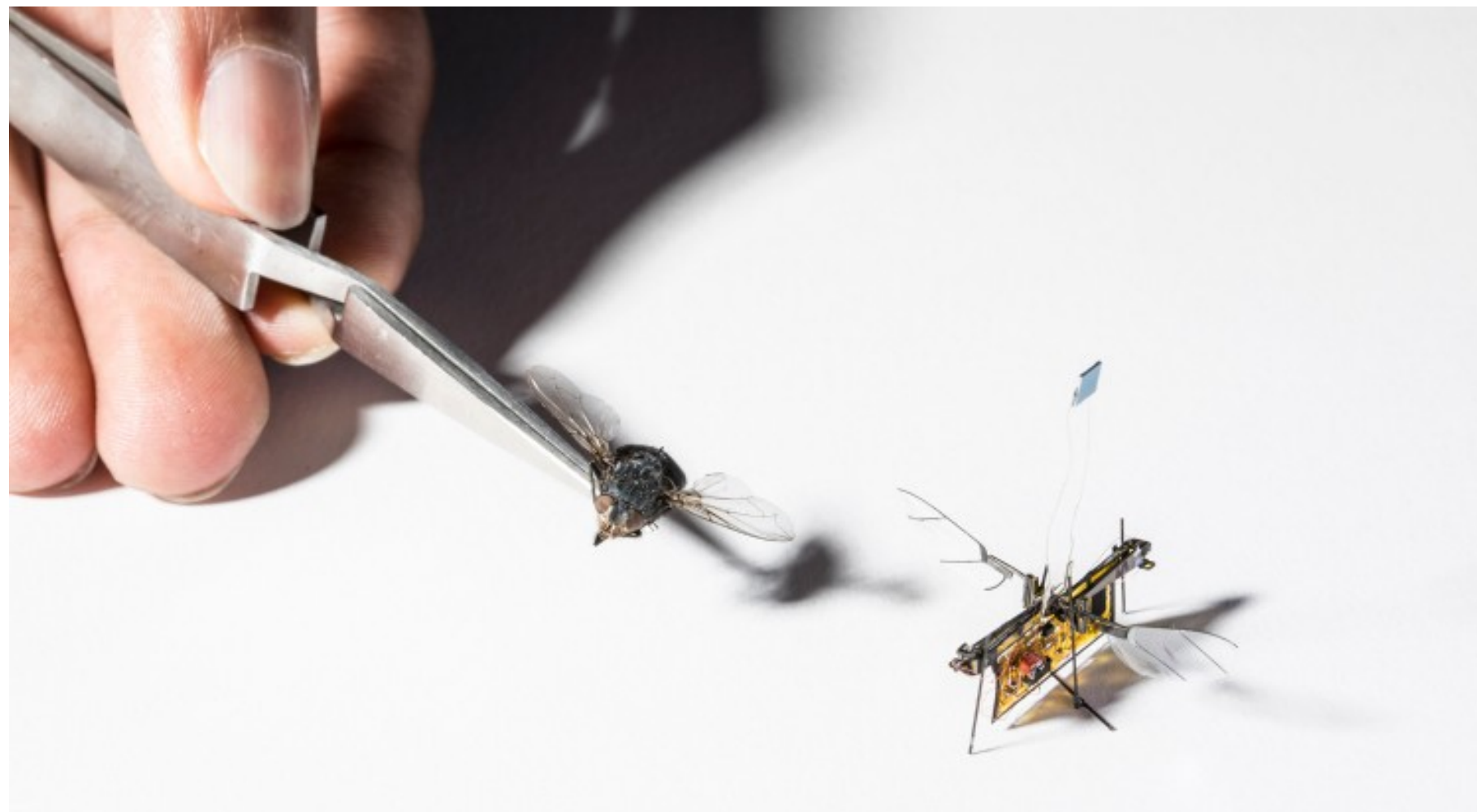
- ▶ **Autonomous systems as a proxy for complex systems, which might have positive societal impact**



*Autonomy for safer and efficient mobility (Motional)*



*Autonomous robots for space exploration (Pavone et al.)*



*Roboflies to monitor environments (Fuller et al.)*



*UAVs for search and rescue tasks (Scaramuzza et al.)*



# Need new tools to model and solve complex systems design optimization problems

- **Societal impact** of new technologies depends on their **joint design** with **existing systems**



*Intermodal mobility networks (NASA UAM)*



*Networks of tankers (Signal Ocean)*

*Example - Autonomy: **Heaven** or **hell**?*

*30% of the cars would be enough*

*First- and last-mile mobility could make **public transit** more **convenient** and **attractive***

*More **affordable**, **sustainable***



***Your Uber Car Creates Congestion. Should You Pay a Fee to Ride?**(New York Times)*



**Data Centers on Wheels: Emissions From Computing Onboard Autonomous Vehicles**

Soumya Sudhakar , Vivienne Sze , and Sertac Karaman , Massachusetts Institute of Technology, Cambridge, MA, 02139, USA

**Single components** are slowly well understood, but we still lack a (*formal* and *practical*) theory for the **task-driven co-design** of **complex systems**



# Agenda

## ▶ **Motivation**

- *New challenges of engineering design*
- *Motivation from autonomy and mobility*
- *Desiderata for co-design*

## ▶ **Monotone Co-Design**

- *Modeling design problems*
- *Examples across domains*
- *Design queries and optimization*
- *From autonomy to mobility systems*

## ▶ **Strategic interactions**

- *Game theory to deal with strategic interactions*
- *Partial order games*

## ▶ **Outlook on future research**

*Website containing all papers and more pointers:*

<https://gioele.science>

Driven by **societal challenges**, I develop **efficient computational tools** to automate the **formulation and solution** of **large, complex system design problems**



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# The vision of automated system co-design

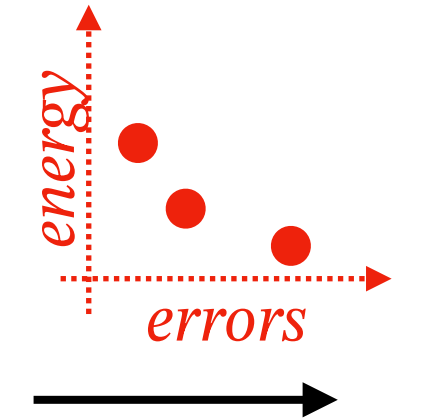
**minimize**  
**(resources usage)**  
**subject to**  
**(functionality constraints)**

## *Autonomy co-design*

task

robot autonomy, physics

components, algorithms



task specification

multi-domain knowledge

design options

“automated designer”

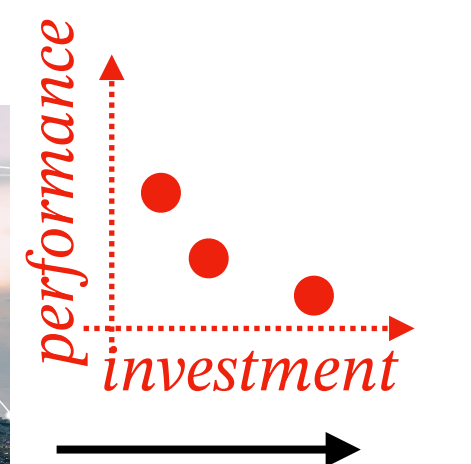
optimal  
design(s)

demand

networks, operations, infrastructure

mobility services, policies

## *Mobility co-design*





# Autonomy as the frontier of complexity for the co-design of complex systems

**A fleet of autonomous vehicles**



=

	hardware	software	behavior	coordination
actuation	sensing	localization	planning	invasivity
computation	control	interaction	learning	liability
	perception	mapping		regulations
energetics	communication		infrastructure	

**OMG!**

So many **components** (hardware, software, ...),  
and **choices** to make!

Nobody understands the **whole** thing!

We forget why we made **choices**, and we are afraid to  
make **changes** (high failure cost).

We need **faster** design cycles, **nimbler** execution.

*anthropomorphization  
of 21st century  
engineering malaise*



“My dear, it’s simple: you lack  
a theory of **co-design!**”

**Formal**

**Quantitative**

**Intellectually tractable**



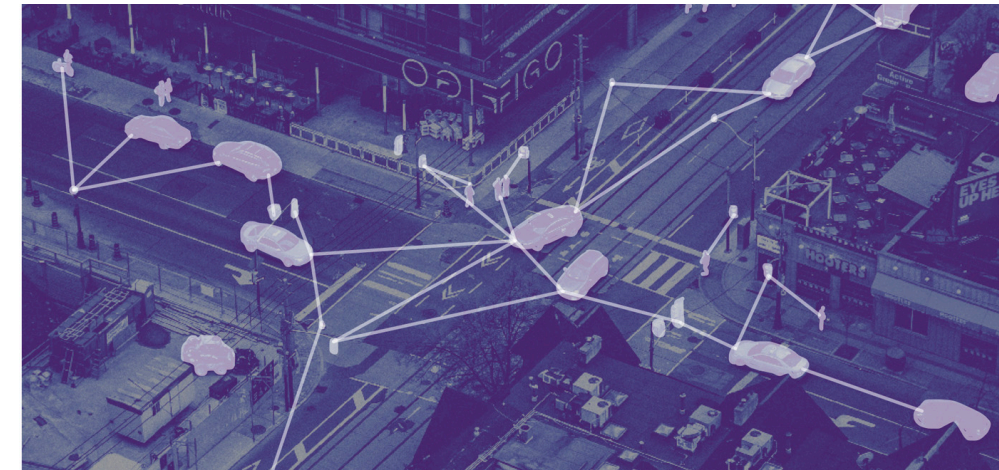
# Your system is just a component in another person's system

*Infrastructure level*



*Optimal infrastructure choices*

*Service level*



*Optimal deployment*

*Platform level*



*Choice of components*

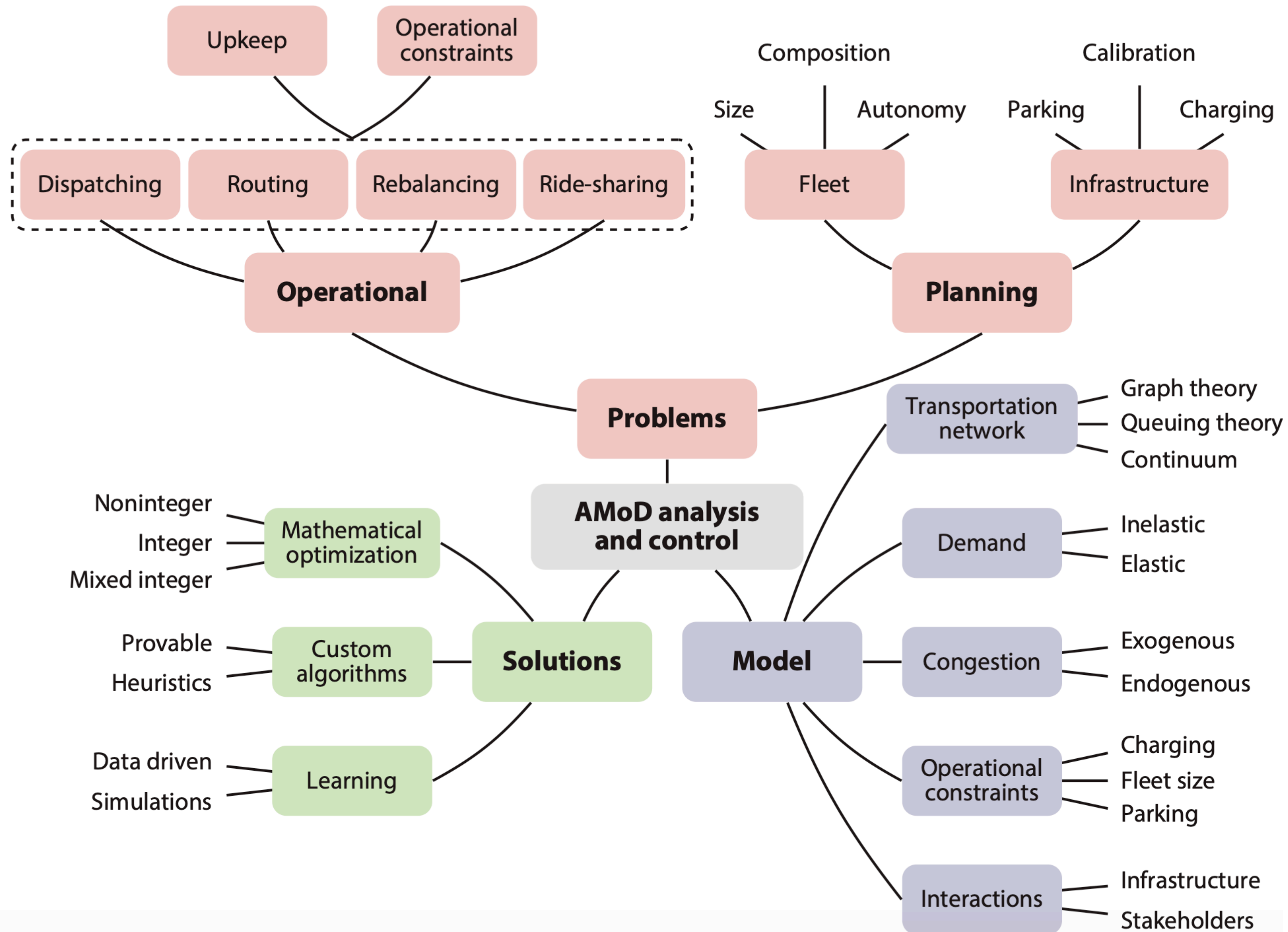
*Subsystem level*



*Single component design*



# Your system is just a component in another person's system





# Complex systems typically feature multi-stakeholders interactions





# Challenges for automated co-design of complex systems

## Complexity when designing complex systems



### Large systems

- Many components, scales
- Heterogeneous natures
- Multiple objectives

### Strategic interactions

- Many agents
- Heterogeneous interactions
- Conflicts/collaborations

**A fleet of autonomous vehicles**



=

software	behavior	coordination	
hardware			
actuation	localization	planning	invasivity
sensing	control	interaction	learning
computation	perception	mapping	regulations
energetics	communication	infrastructure	





# Desiderata for the automation of complex systems co-design

- ▶ **Formal, domain-independent**
- ▶ **Computationally tractable**
  - Need to compute solutions efficiently
- ▶ **Compositional, hierarchical**
  - My system is a component of somebody else's system
- ▶ **Collaborative**
  - Pooling knowledge from experts across fields.
- ▶ **Intellectually tractable**
  - Not exclusively accessible to system architects
- ▶ **Continuous**
  - Design is not static: it should be reactive to changes in goals and contexts



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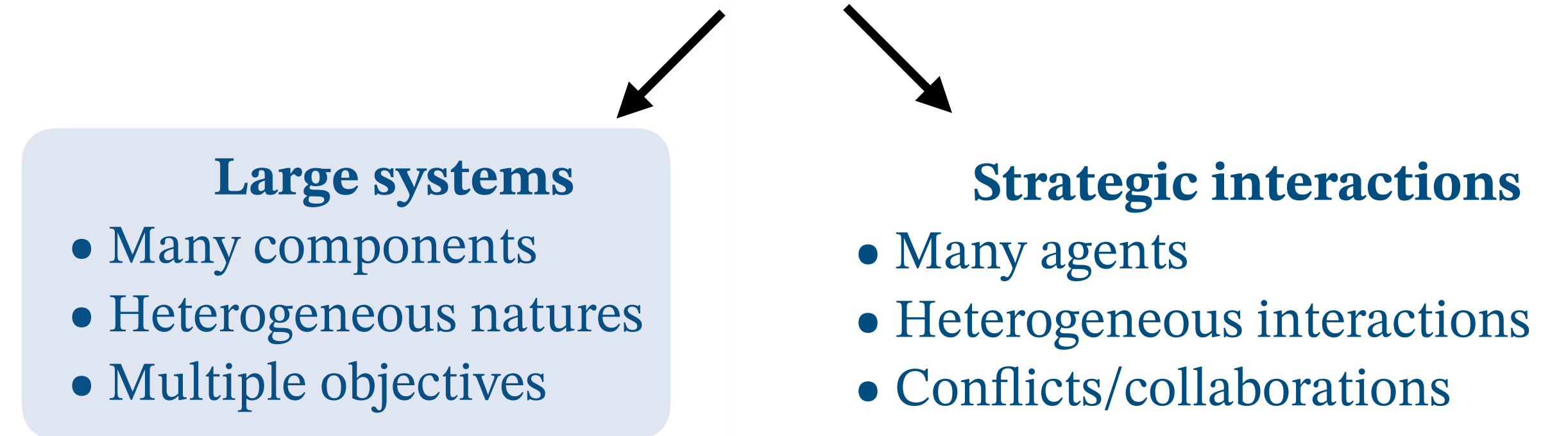
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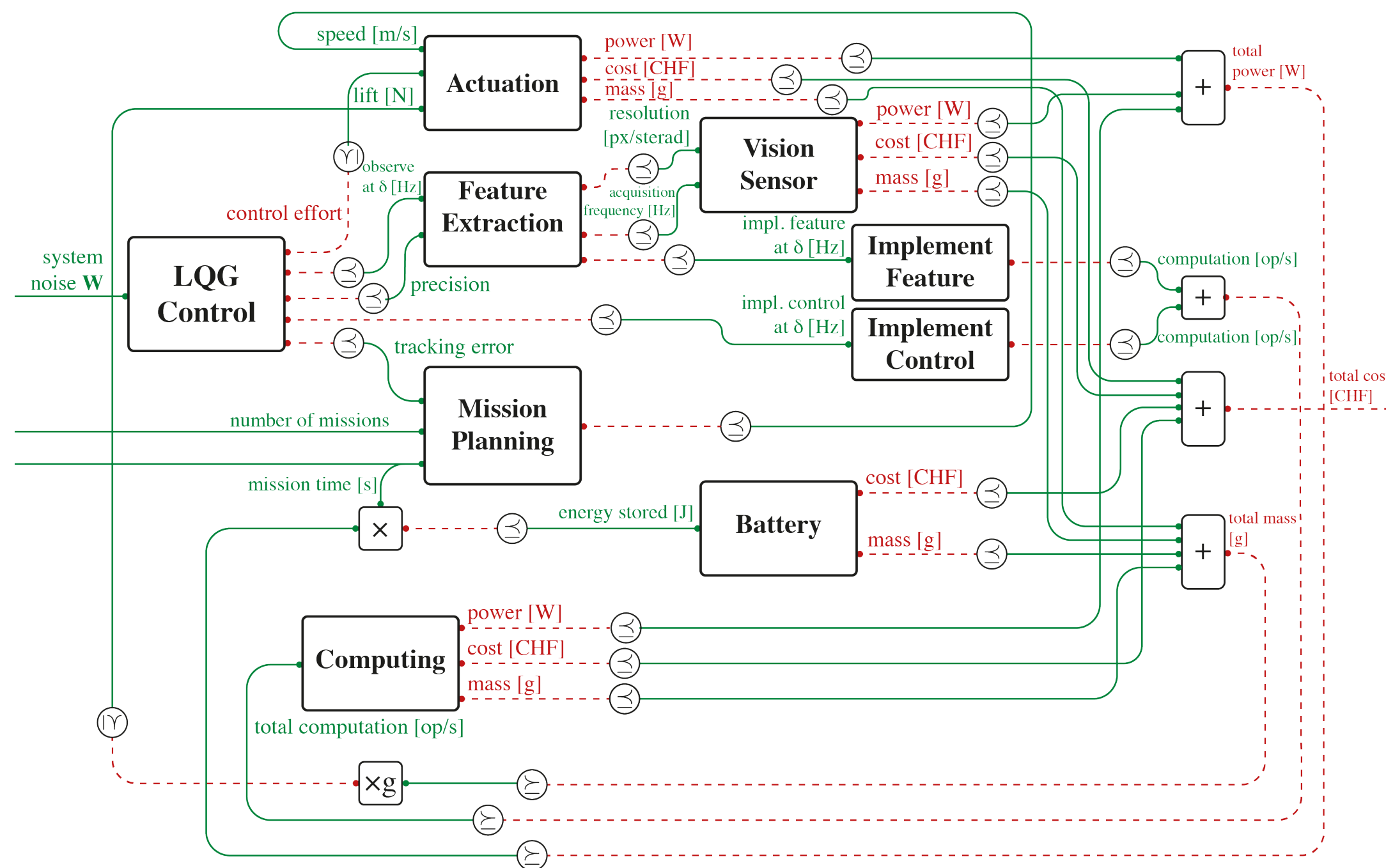




# A new approach to multi-disciplinary engineering “co”-design

- ▶ A new approach to **collaborative, computational, compositional, continuous** design designed to work **across fields** and **across scales**.
- ▶ Leverages **domain theory, applied category theory, and optimization**
- ▶ Roadmap:
  - Defining “**design problems**” for **components**.
  - Modeling **co-design constraints** in a complex **system**.
  - **Efficient** solution to design queries.

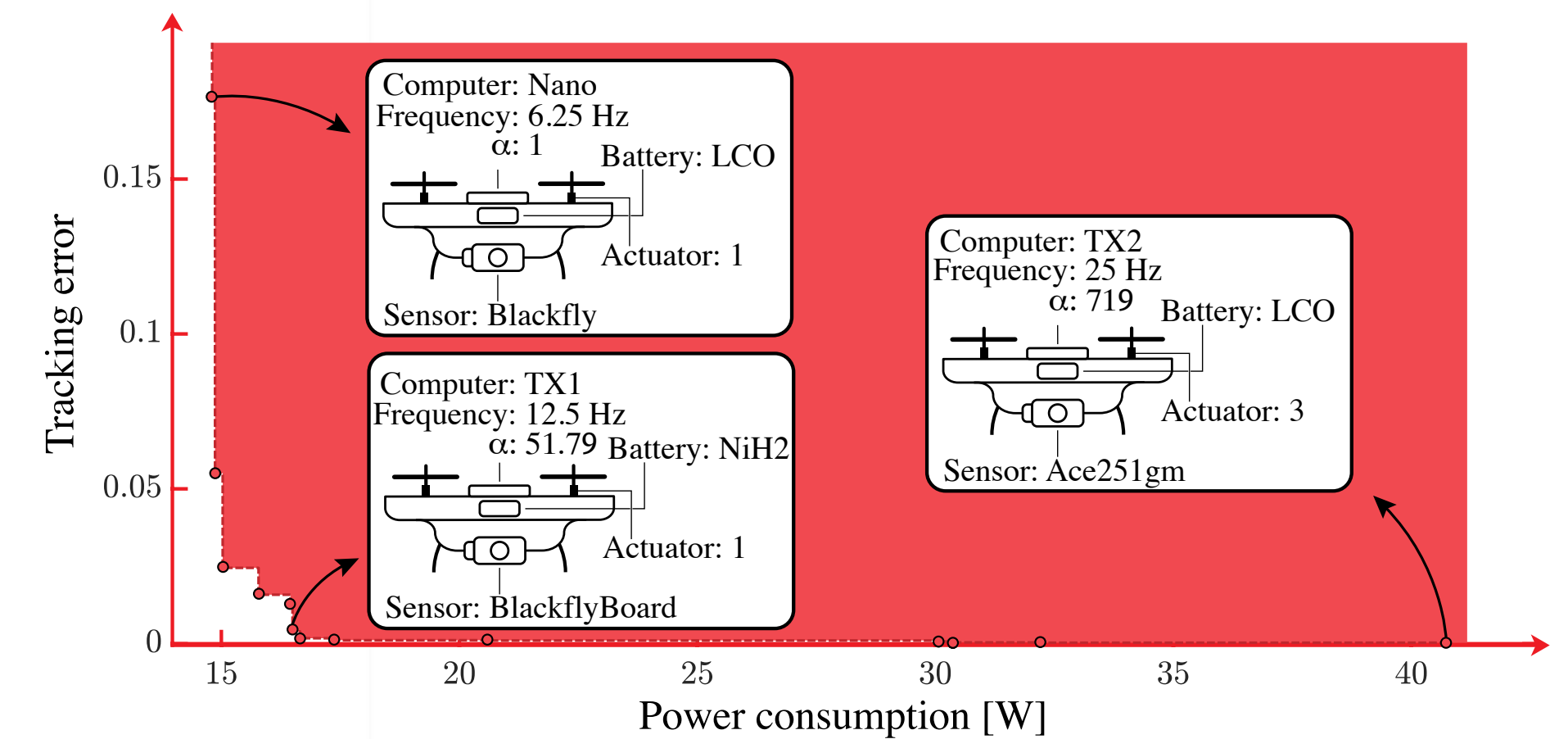
“Co-design diagram”



optimization  
for a task



Pareto front of optimal designs





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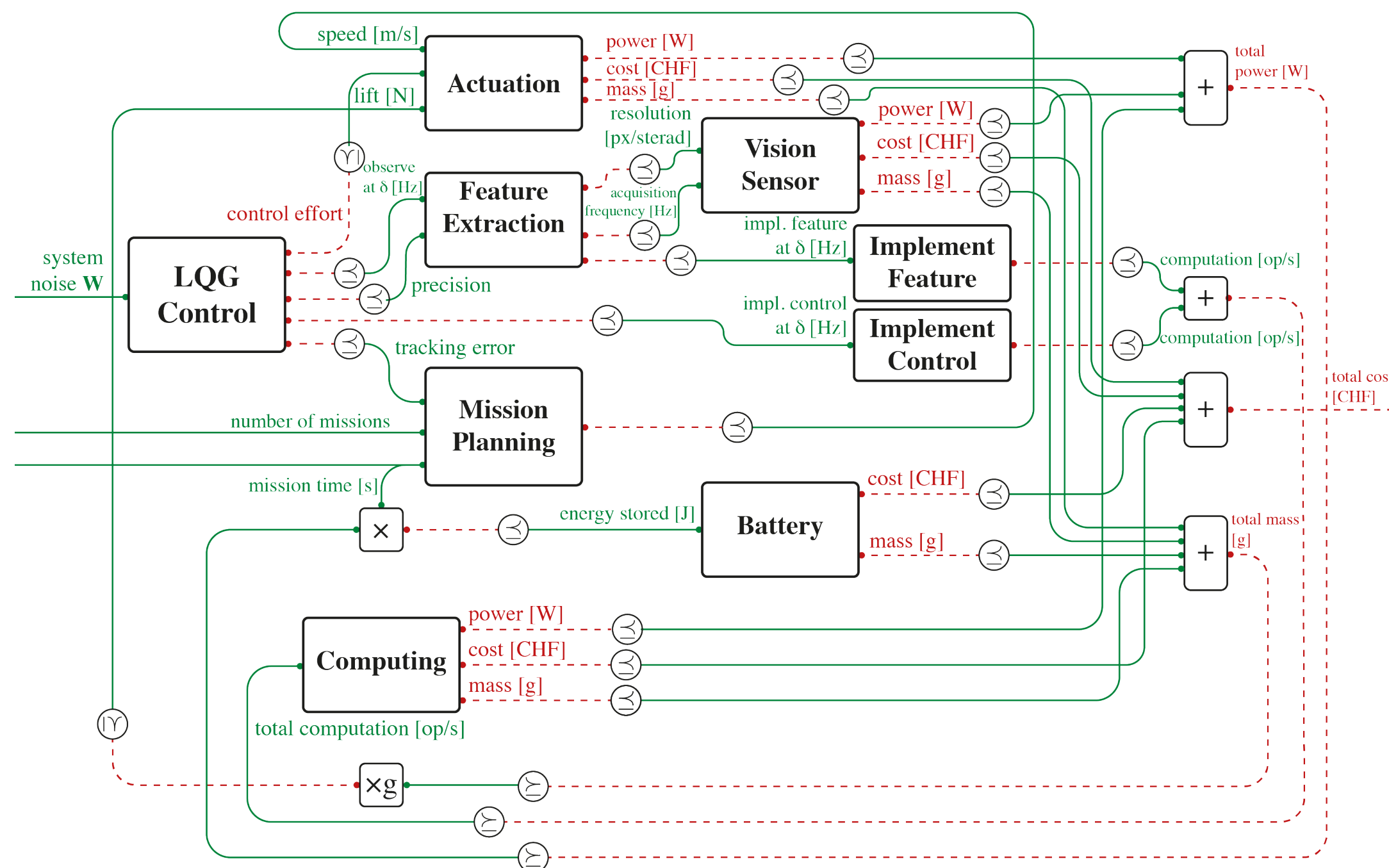
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*a “pro” box*



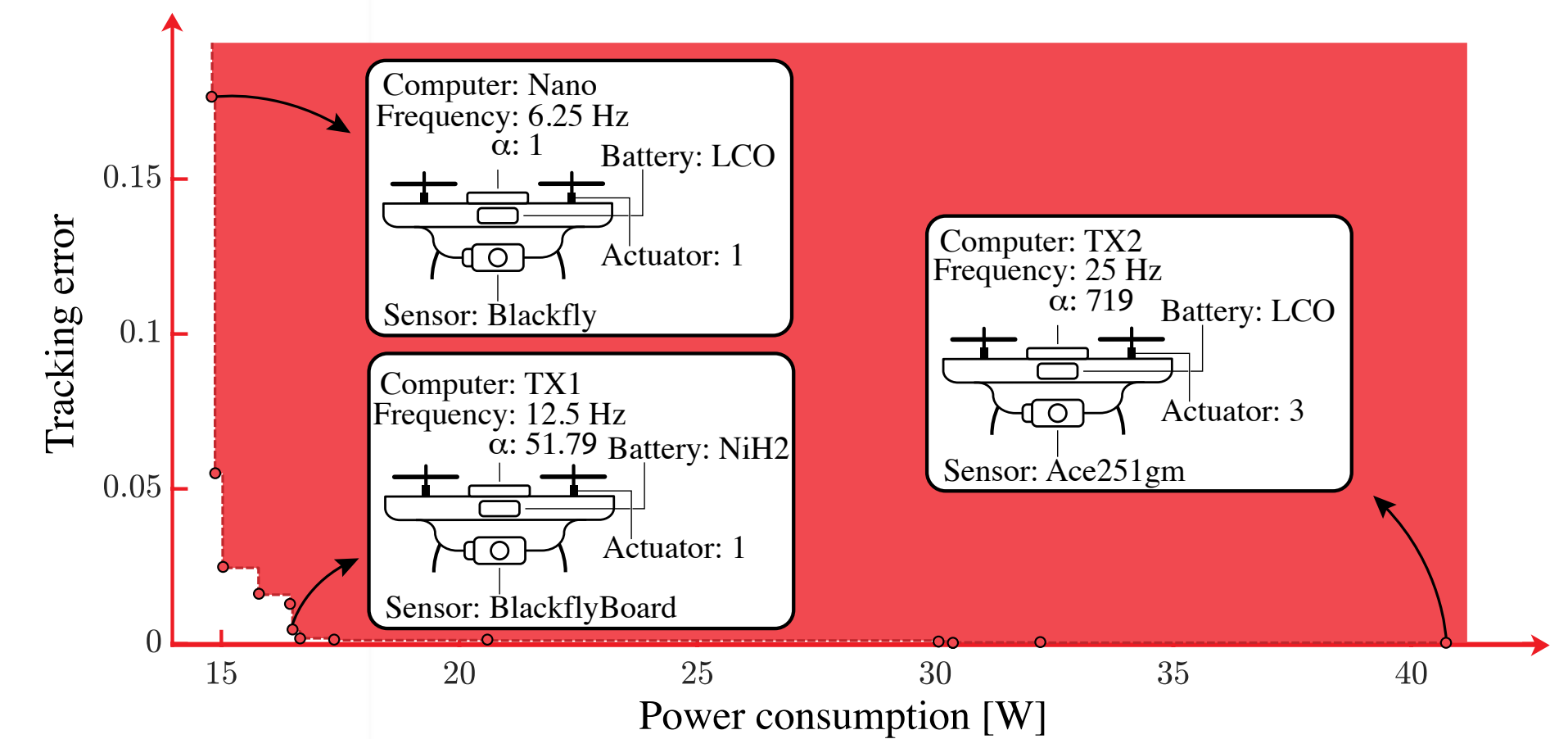
Access the book at:  
<https://bit.ly/3qQNrdR>

*“Co-design diagram”*



*optimization for a task*

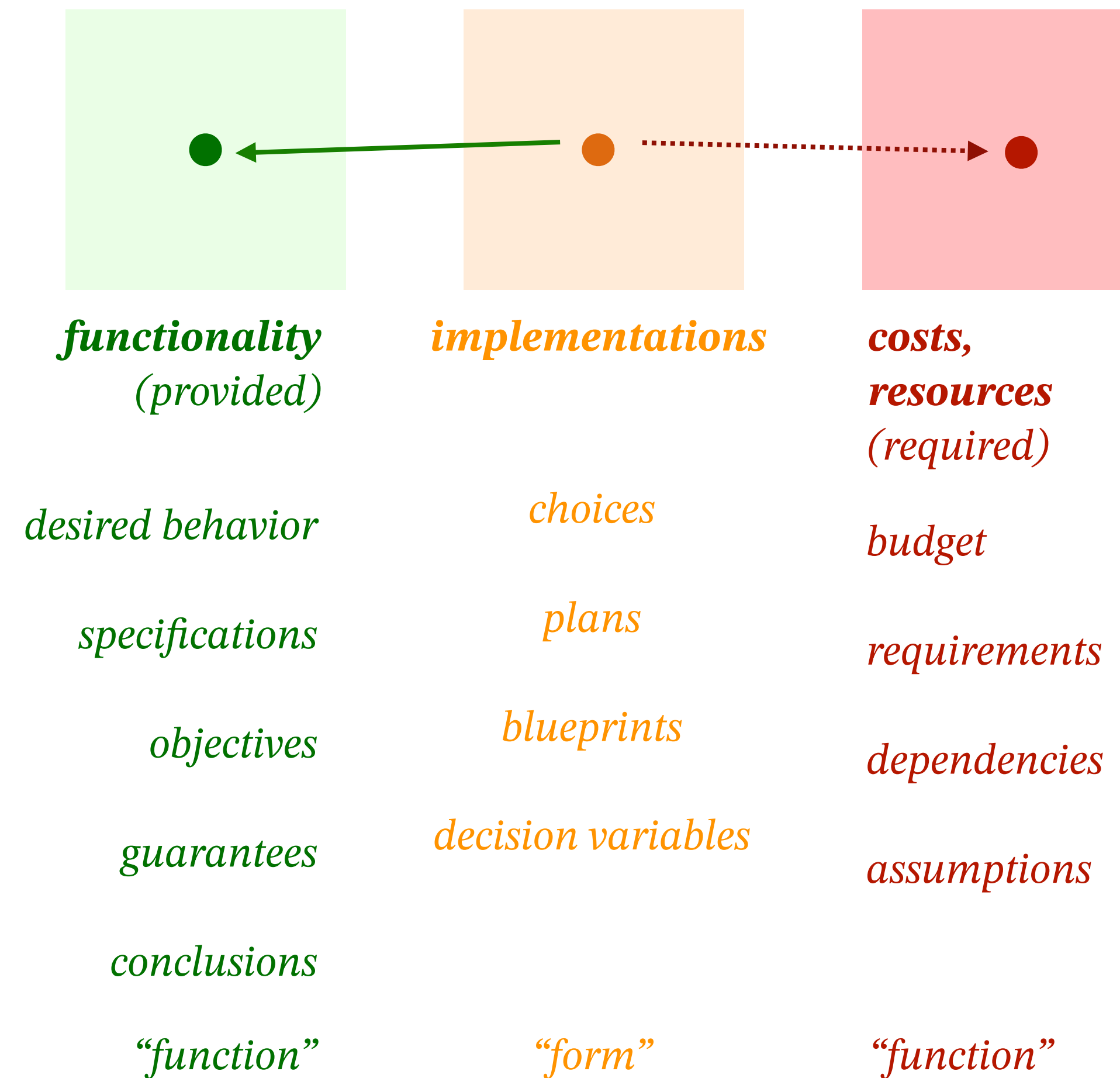
*Pareto front of optimal designs*





# An abstract view of design problems

- ▶ Across fields, design or synthesis problems are defined with **three spaces**:
  - **implementation space**: the **options** we can choose from;
  - **functionality space**: what we need to **provide/achieve**;
  - **requirements/costs space**: the **resources** we need to have available;

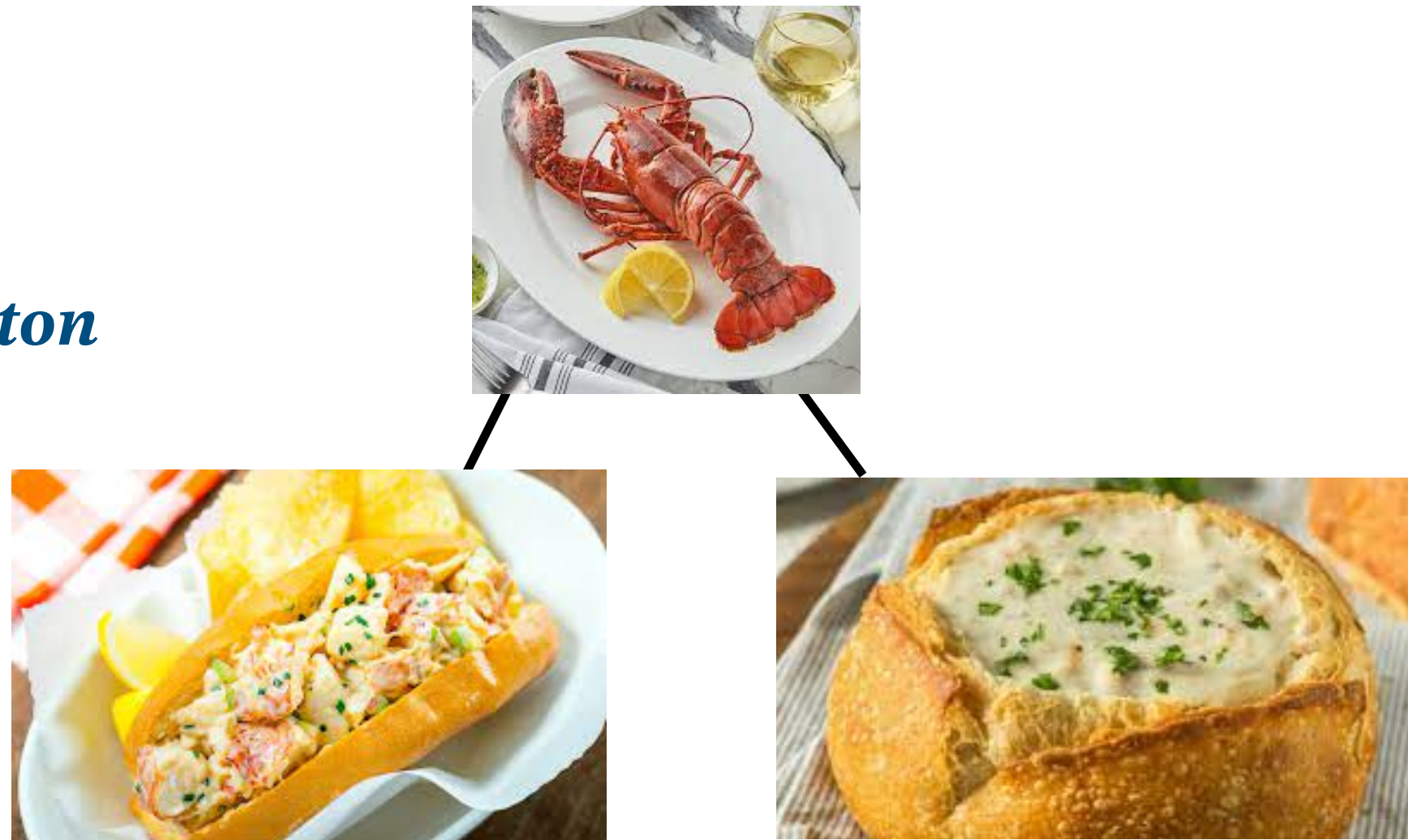




# Partially ordered sets model trade-offs, across fields

- ▶ Posets model standard costs in engineering  $\langle \mathbb{R}_{\geq 0}, \leq \rangle$ ,  $\langle \mathbb{N}, \leq \rangle$
- ▶ ... but also enable **richer** cost structures, with **incomparable** elements

*A poset of food preferences in Boston*



*A poset of positive-definite matrices*

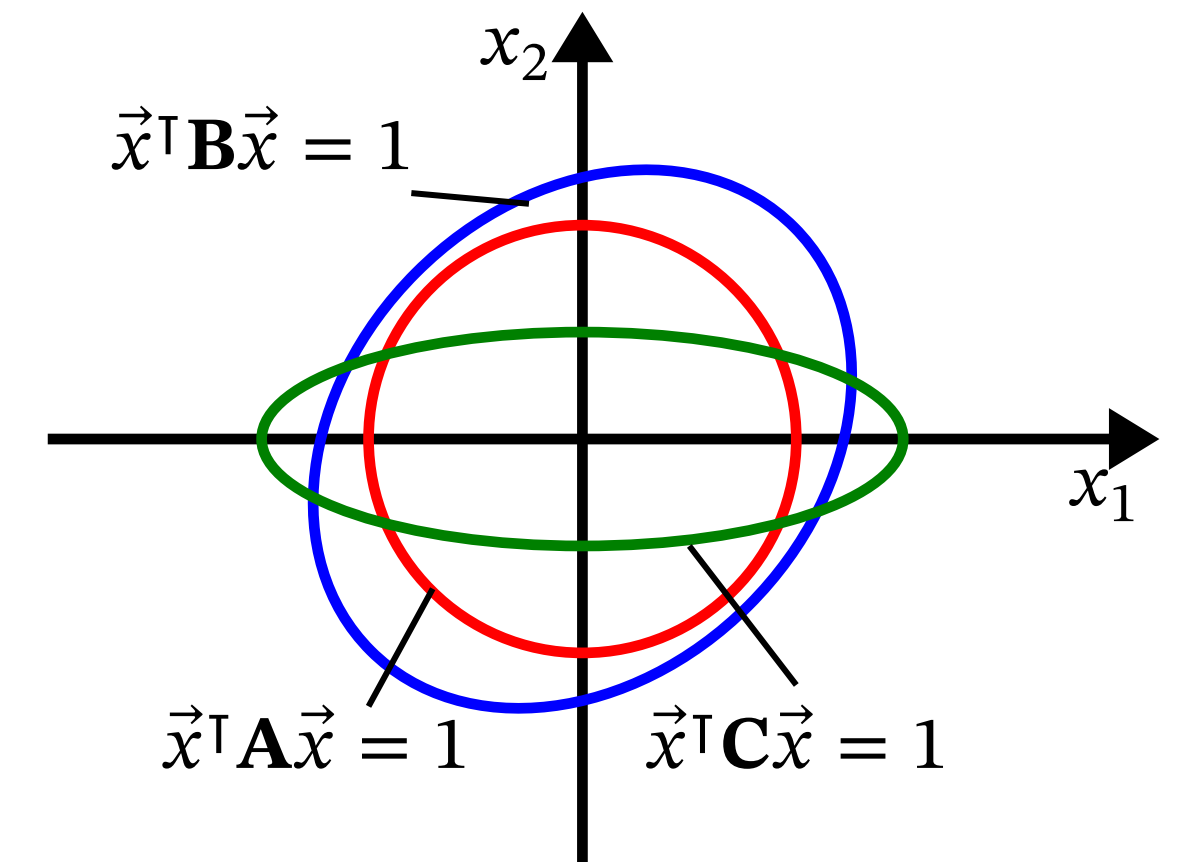
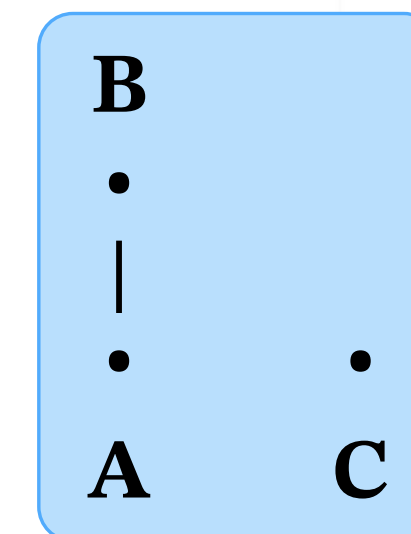
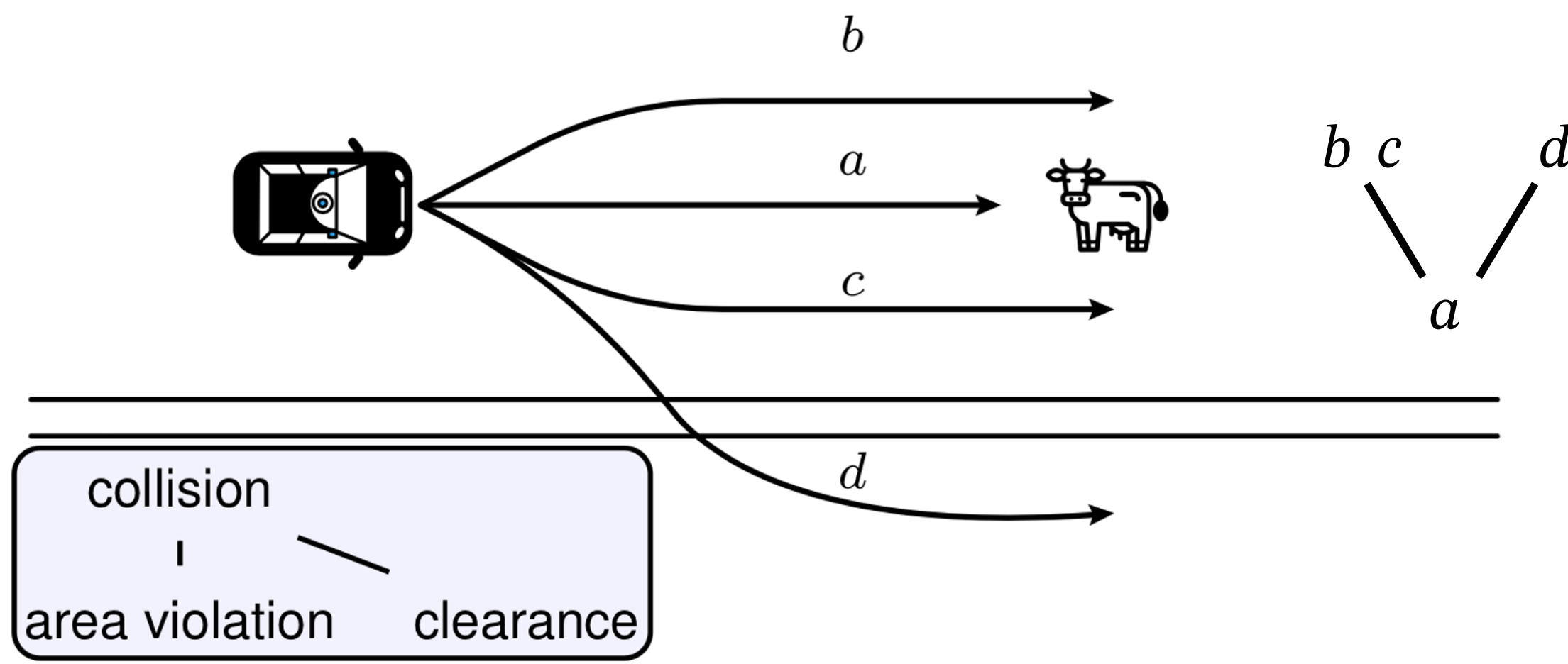
$$\mathbf{A} \leq_{\text{PDM}(n)} \mathbf{B}$$

---


$$\vec{x}^T \mathbf{A} \vec{x} \leq \vec{x}^T \mathbf{B} \vec{x} \quad \forall \vec{x} \in \mathbb{R}^n$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3/4 & -1/8 \\ -1/8 & 3/4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$

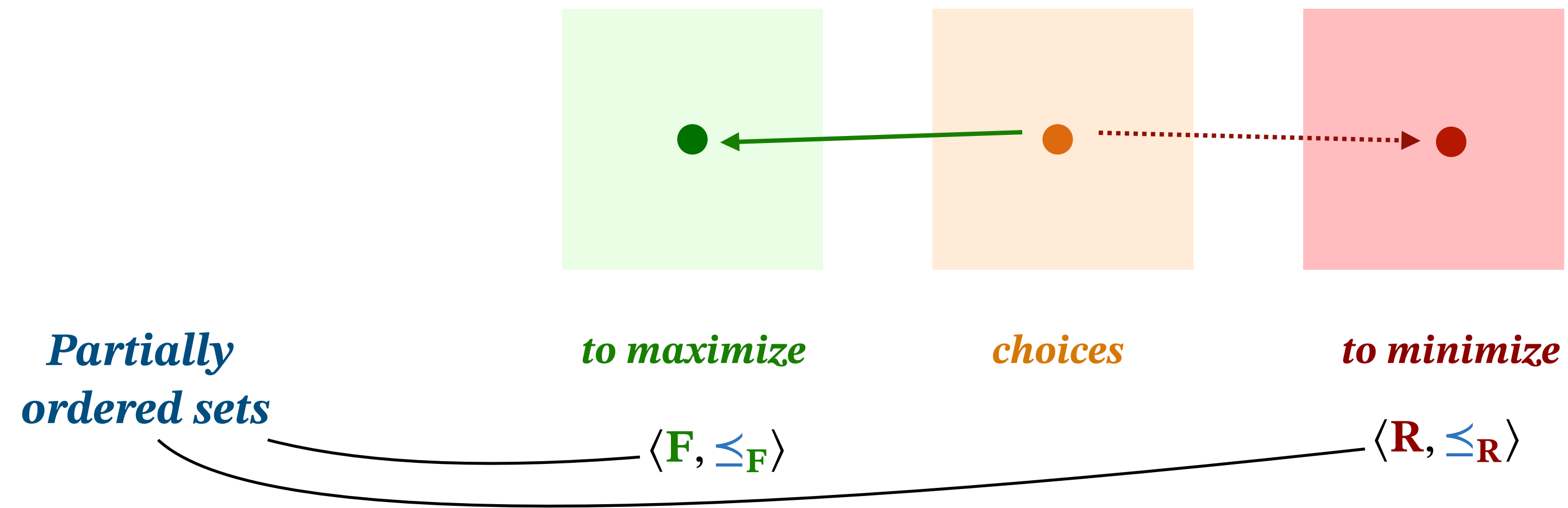
*Posets of rules, which induce priorities over behaviors*





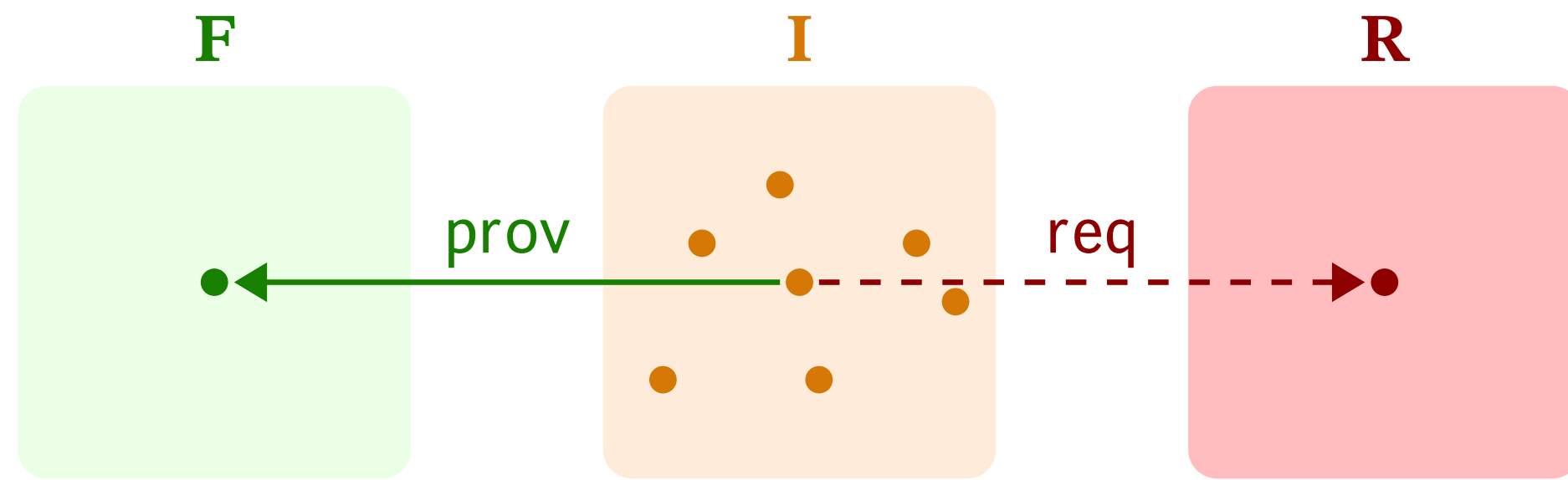
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# Transparent vs black-box models

- ▶ The “Design Problems with Implementations” model is a “transparent” model:



- ▶ **DP** model: **direct feasibility relation** between functionality and resources (“black box”) as a monotone map:



$$\mathbf{d} : \mathbf{F}^{\text{op}} \times \mathbf{R} \rightarrow_{\text{Pos}} \mathbf{Bool}$$

$$\langle f^*, r \rangle \mapsto \exists i \in \mathbf{I} : (f \leq_{\mathbf{F}} \text{prov}(i)) \wedge (\text{req}(i) \leq_{\mathbf{R}} r)$$

... a “boolean profunctor”

- ▶ **Monotonicity:**

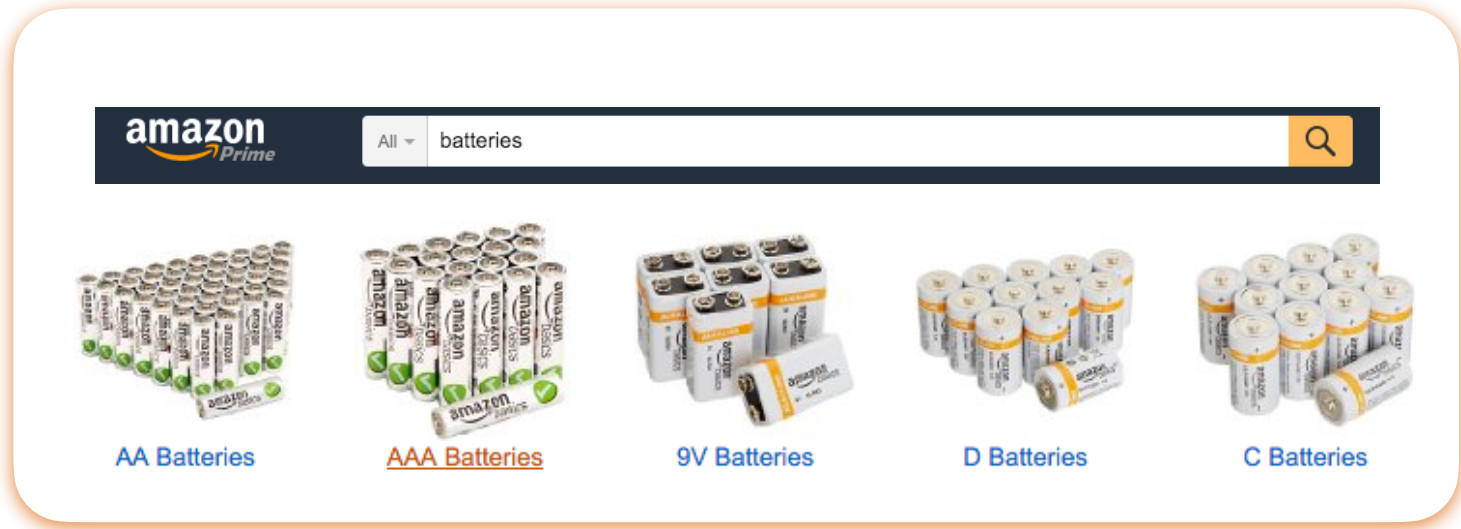
- Lower **functionality** does **not** require **more resources**;
- More **resources** do not provide **less functionality**.





# Co-design enables a rich class of model population techniques

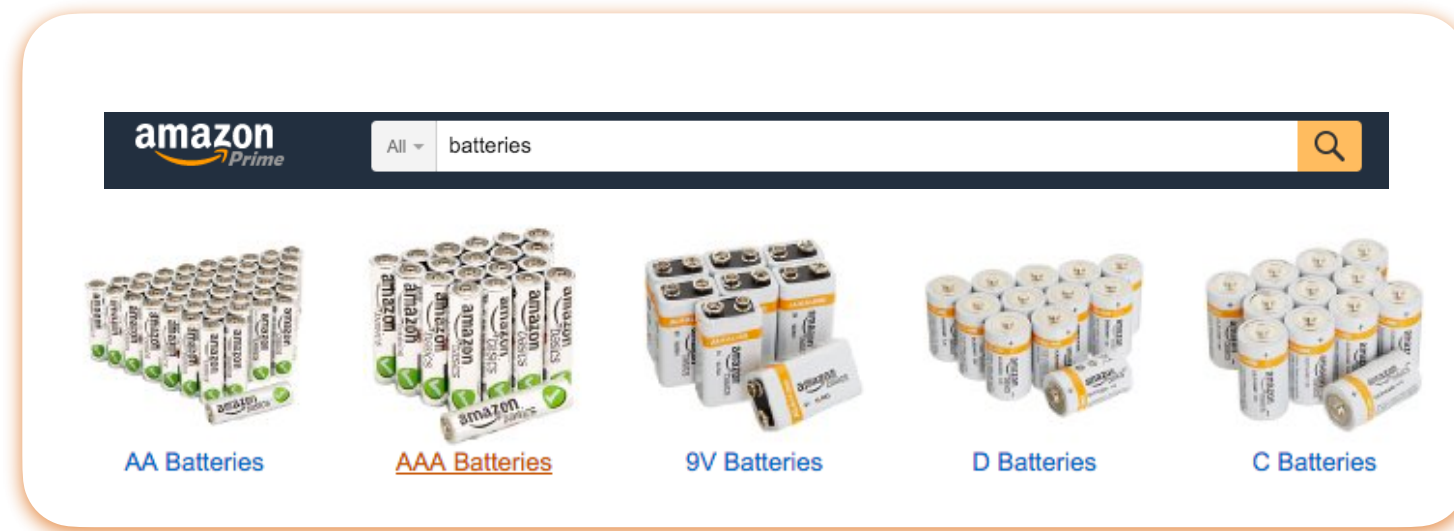
► “Catalogues”: off-the-shelf designs.



	Spark	Phantom 3 Std	Phantom 4 Adv	Phantom 4 Pro	Mavic	Inspire
<b>Flight time</b>	16 mins	25 mins	30 mins	30 mins	27 mins	27 mins
<b>Top Speed</b>	31 mph (50 km/h)	36 mph (58 km/h)	45 mph (72 km/h)	45 mph (72 km/h)	40 mph (65 km/h)	58 mph (94 km/h)
<b>Range</b>	1.2 miles (2 km)	0.6 miles (1 km)	4.3 miles (7 km)	4.3 miles (7 km)	4.3 miles (7 km)	4.3 miles (7 km)
<b>Camera</b>	12-MP stills 1080p video	12-MP stills 2704 x 1520p video	20-MP stills 4K 60fps video	20-MP stills 4K 60fps video	12-MP stills 4K video	20.8-MP stills 4K/5K video
<b>Size</b>	5.6 x 5.6 x 2.1 in (14.3 x 14.3 x 5.5 cm)	13.8 in diagonal (350 mm)	13.8 in diagonal (350 mm)	13.8 in diagonal (350 mm)	13.2 in diagonal (350 mm)	16.8 x 12.5 x 16.7 in (42.7 x 31.7 x 42.5 cm)
<b>Takeoff weight</b>	11.6 oz (330 g)	2.6 lb (1.2 kg)	3 lb (1.4 kg)	3 lb (1.4 kg)	1.6 lb (743 kg)	8.8 lb (4 kg)
<b>Other features</b>	Follow me, Return home, Obstacle avoidance, FPV	Follow me, Return home	Follow me, Return home, Obstacle avoidance	Follow me, Return home, 3 Direction Obstacle avoidance	Follow me, Return home, Obstacle avoidance, folding arms	Obstacle avoidance, Spotlight Pro/Broadcast/Composition mode
<b>Price</b>	US\$499	US\$499	US\$1,349	US\$1,499	US\$999	US\$2,999 (\$6,198 with camera/gimbal)

# Co-design enables a rich class of model population techniques

- ▶ “Catalogues”: off-the-shelf designs.



- ▶ “First-principles”: analytical relations.



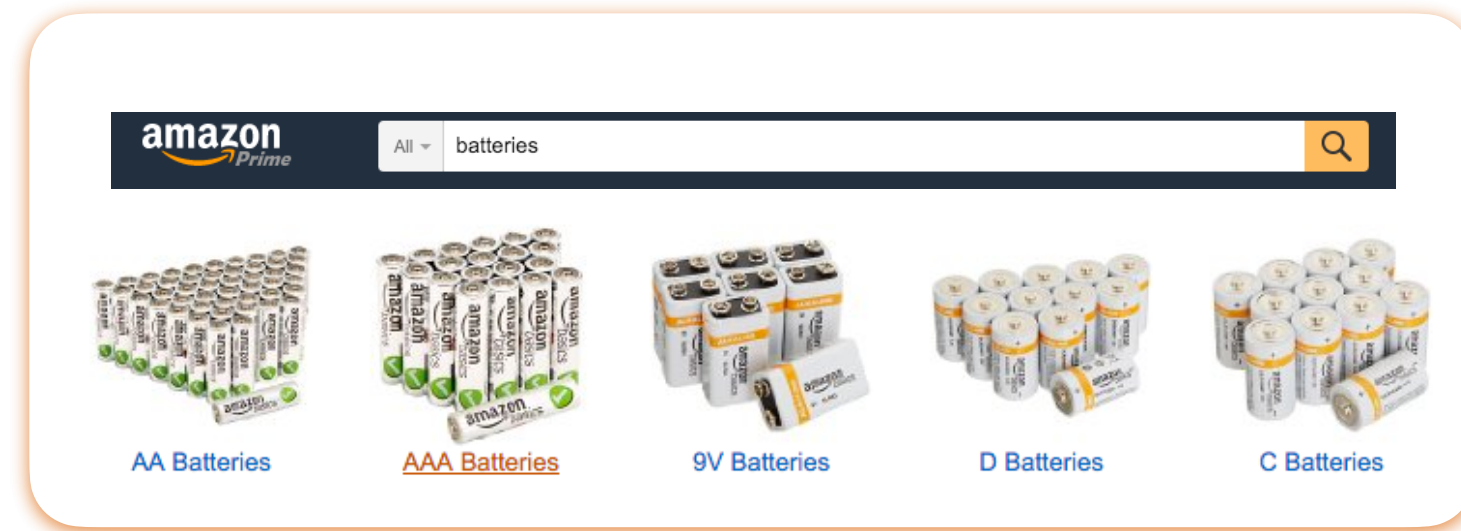
$$\text{mission energy} \geq \text{mission duration} \times \text{power consumption}$$





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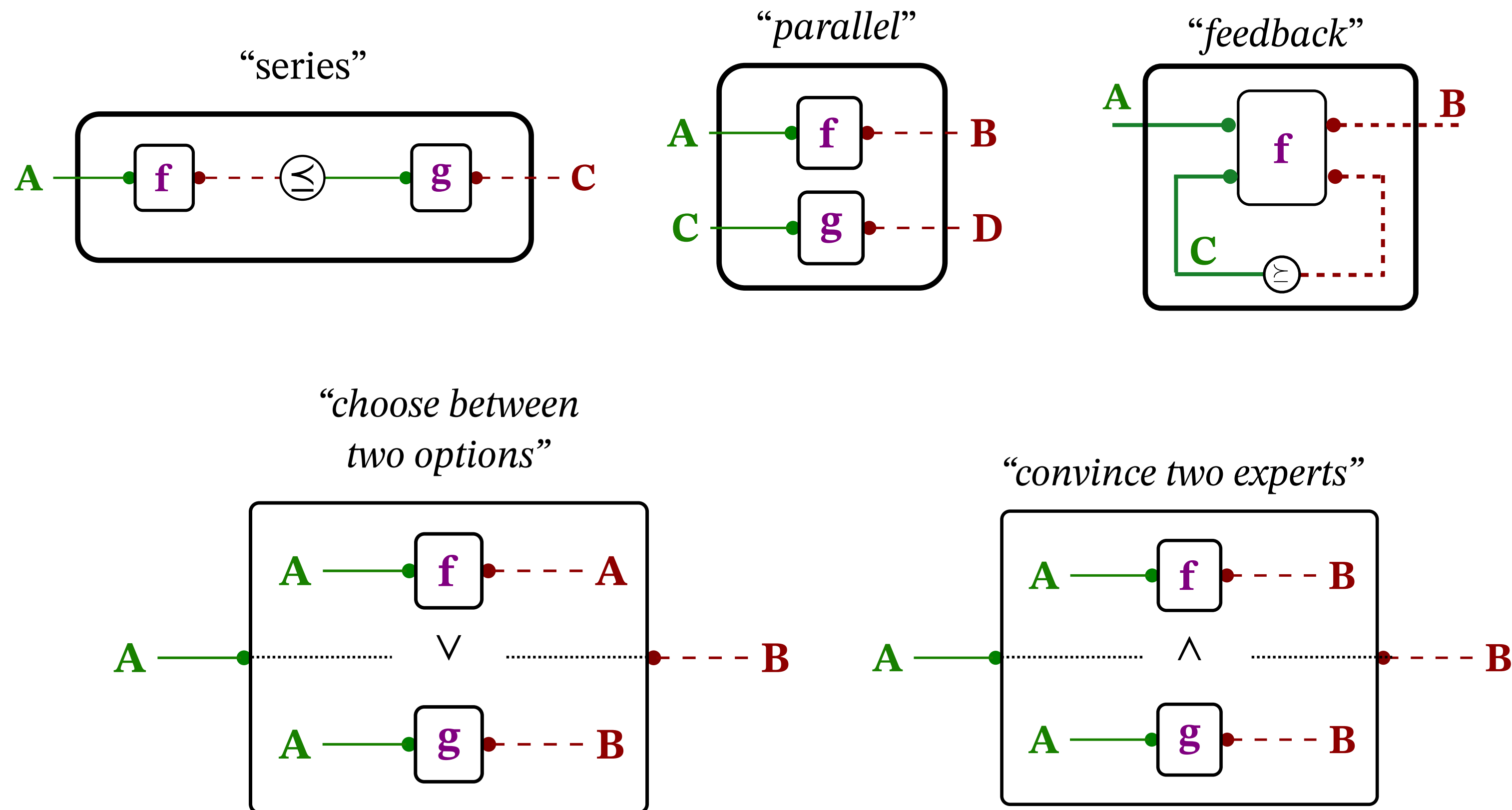


$$\text{mission energy} \geq \text{mission duration} \times \text{power consumption}$$

- ▶ “Data-driven”, “on-demand”

- The optimization will only ask for a **sequence** of data points. The model is constructed **incrementally**.
- Opens the door to **experiments**, black-box **simulations**, solutions of **optimization problems**.

# Design problems can be composed in various ways, preserving properties



- ▶ The **composition** of any two **DPs** returns a **DP** (closure)
- ▶ Very practical tool to **decompose** large **problems** into **subproblems**

*There is a category **DP** which is traced monoidal, and locally posetal*

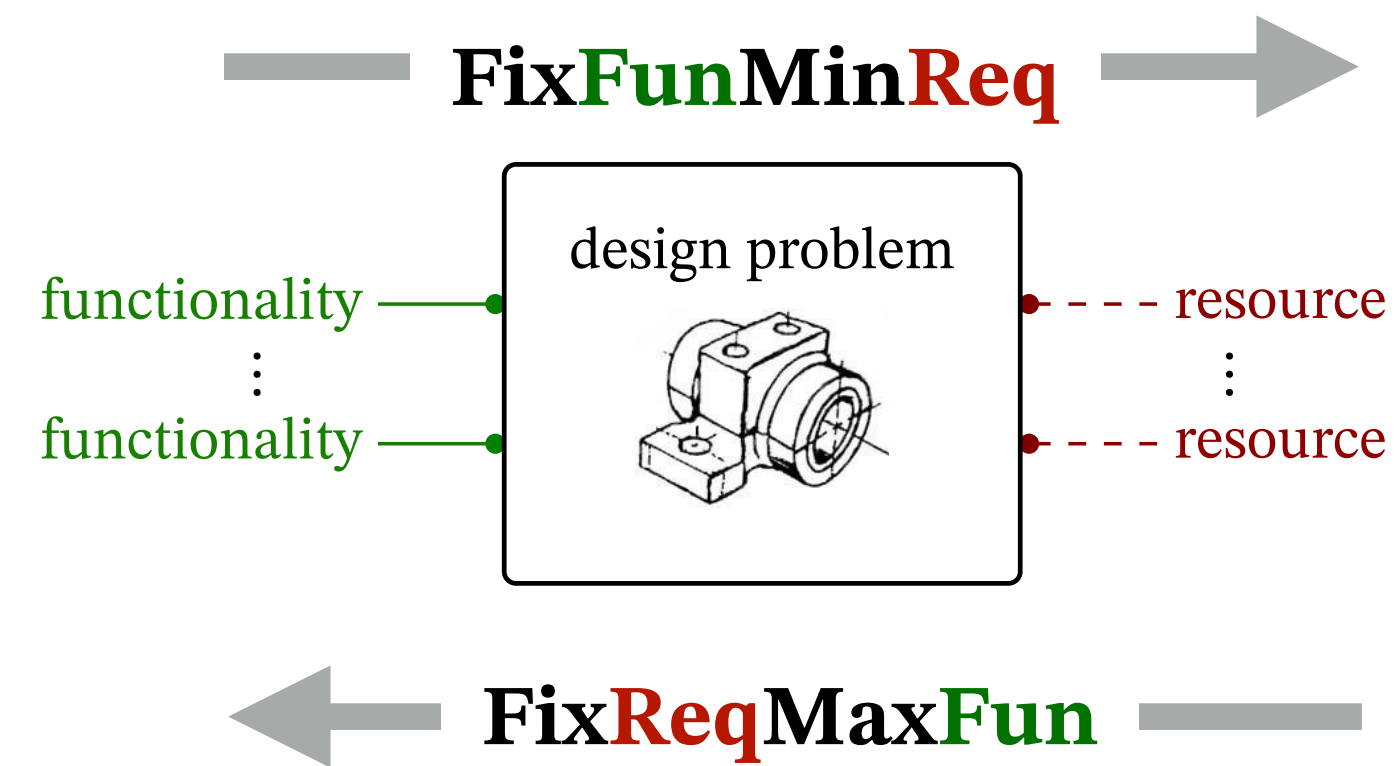
✓ **Formal  
Compositional/hierarchical**



# Multiple queries from the same design problem

- ▶ Two basic design queries are:

Given the **functionality** to be provided,  
what are the **minimal resources** required?

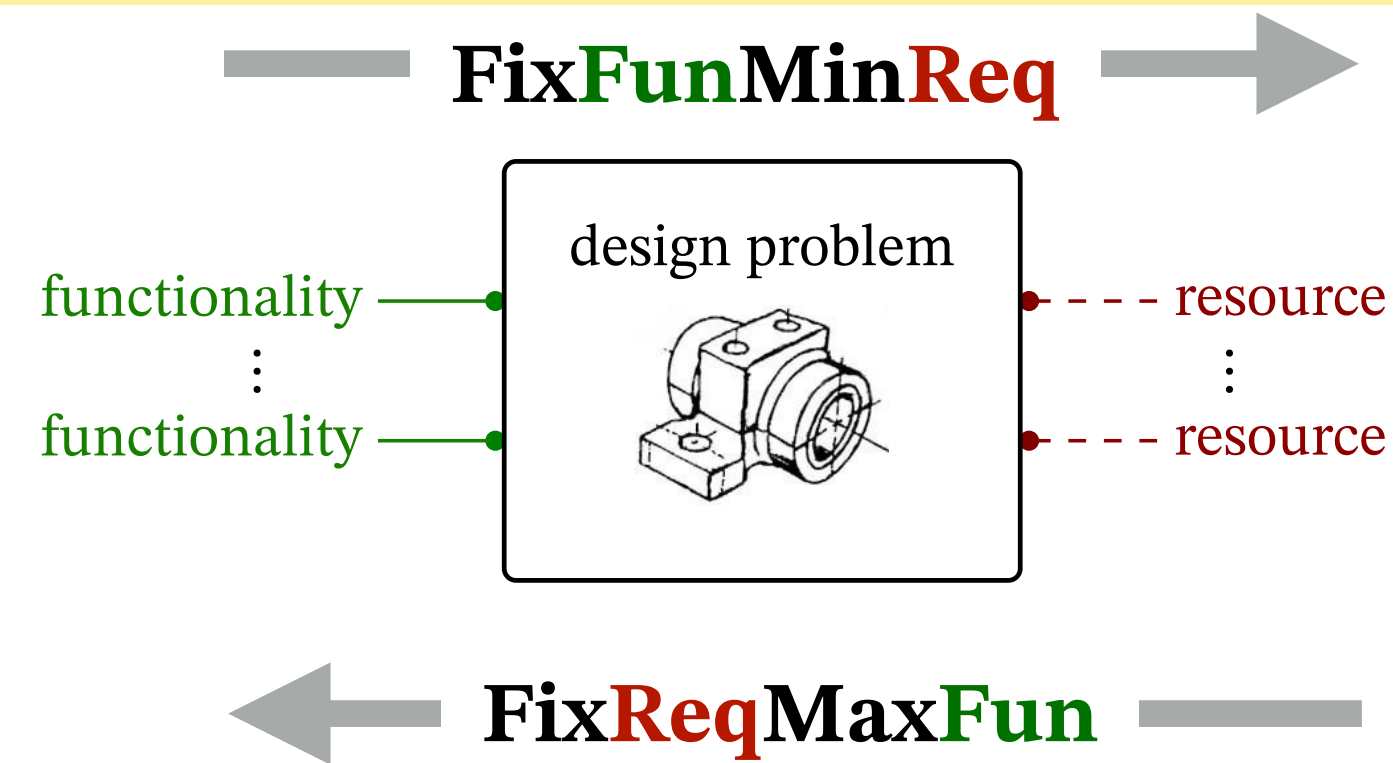


Given the **resources** that are available, what is  
the **maximal functionality** that can be provided?

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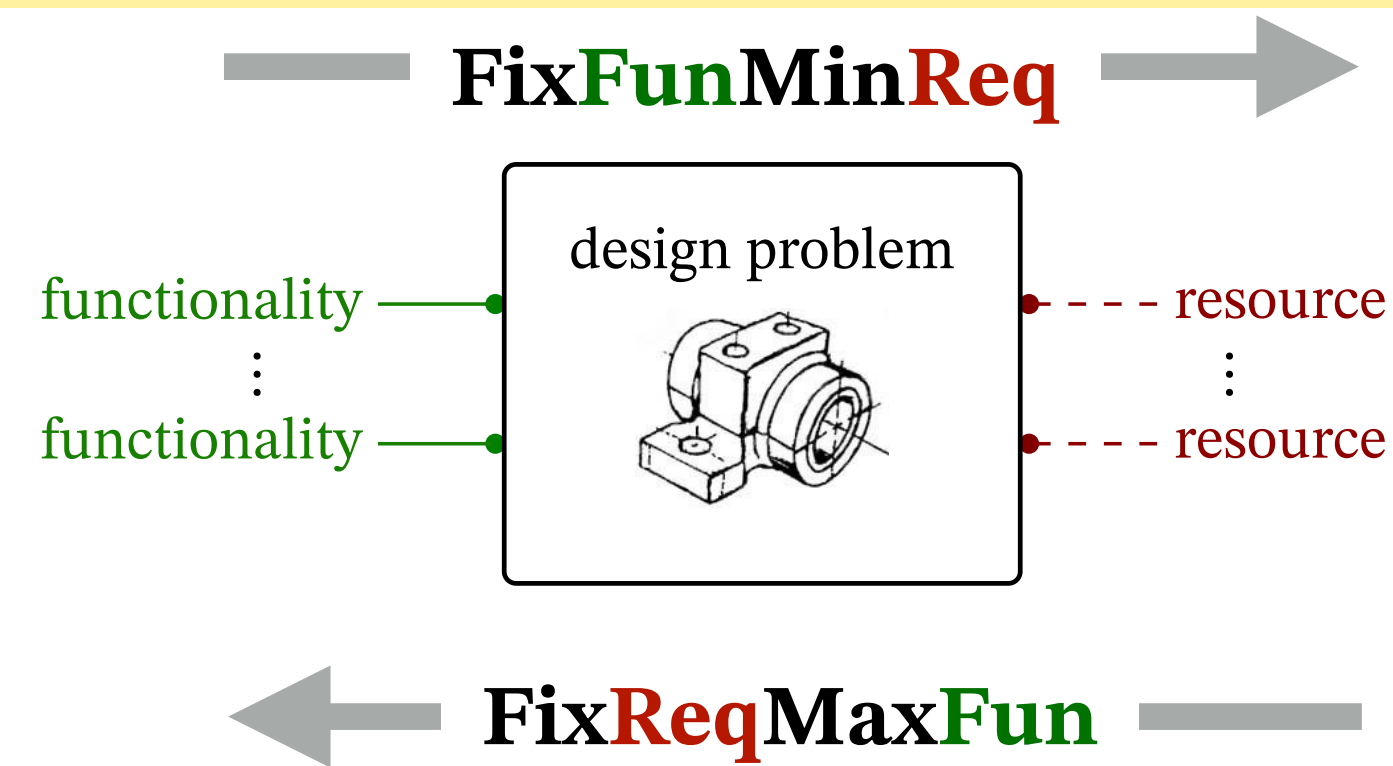
- ▶ The two problems are **dual**



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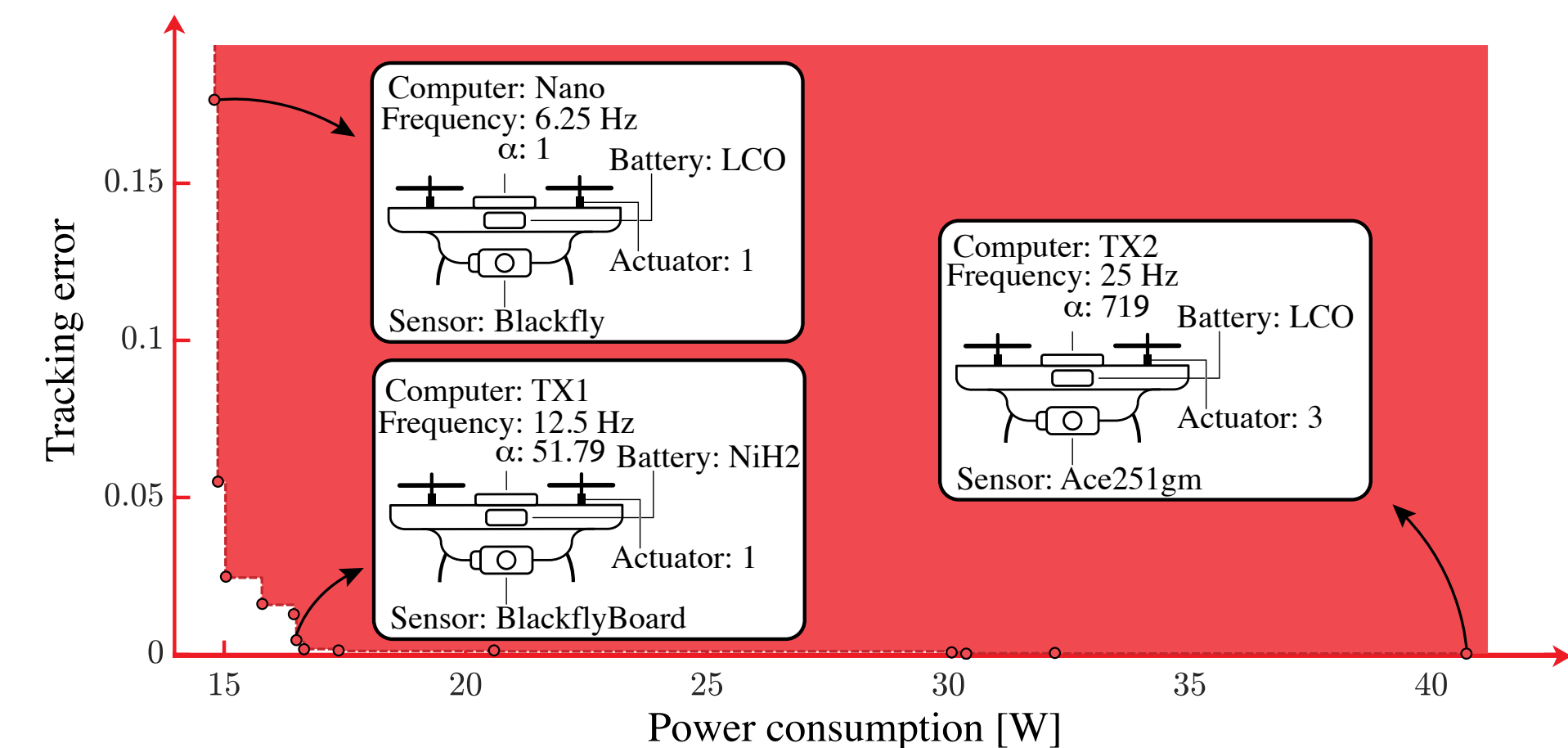
Given the **functionality** to be provided, what are the **minimal resources** required?



Given the **resources** that are available, what is the **maximal functionality** that can be provided?

► We are looking for:

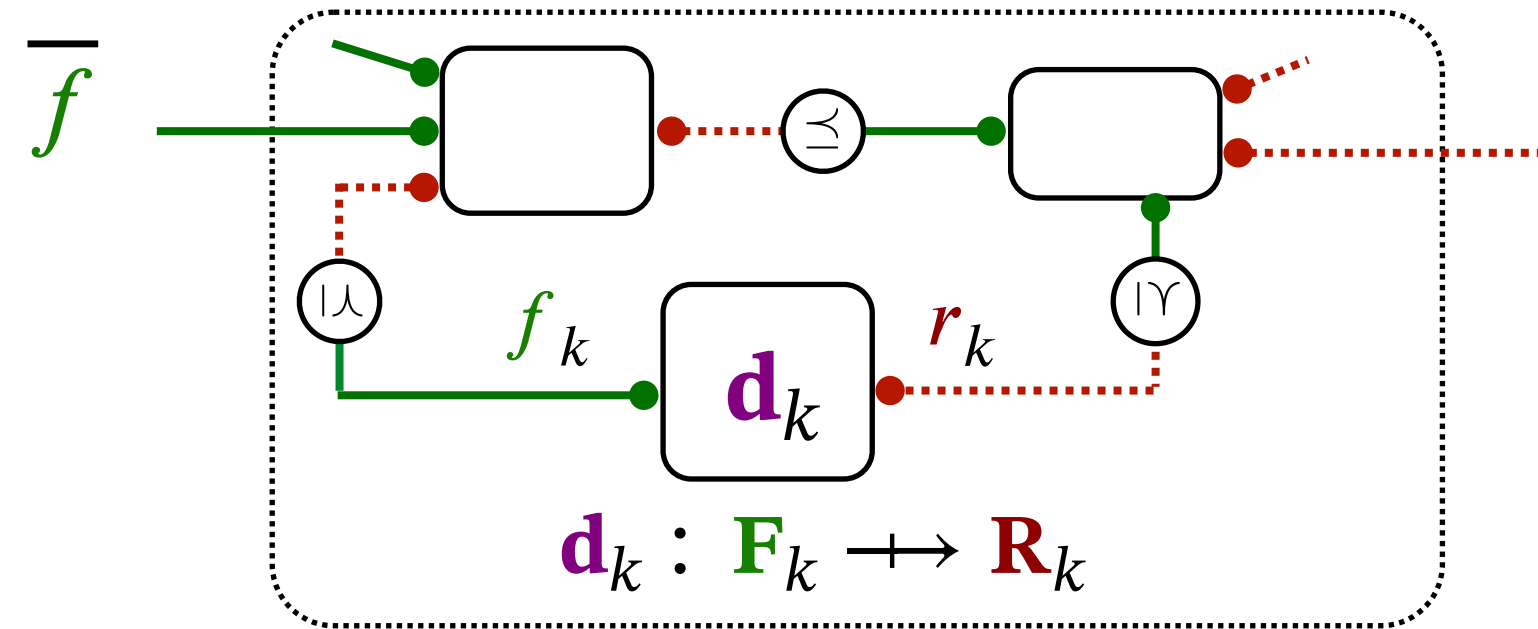
- A map from functionality to **upper sets** of feasible resources:  $h : \mathbf{F} \rightarrow \mathcal{UR}$
- A map from functionality to **antichains** of minimal resources:  $h : \mathbf{F} \rightarrow \mathcal{AR}$



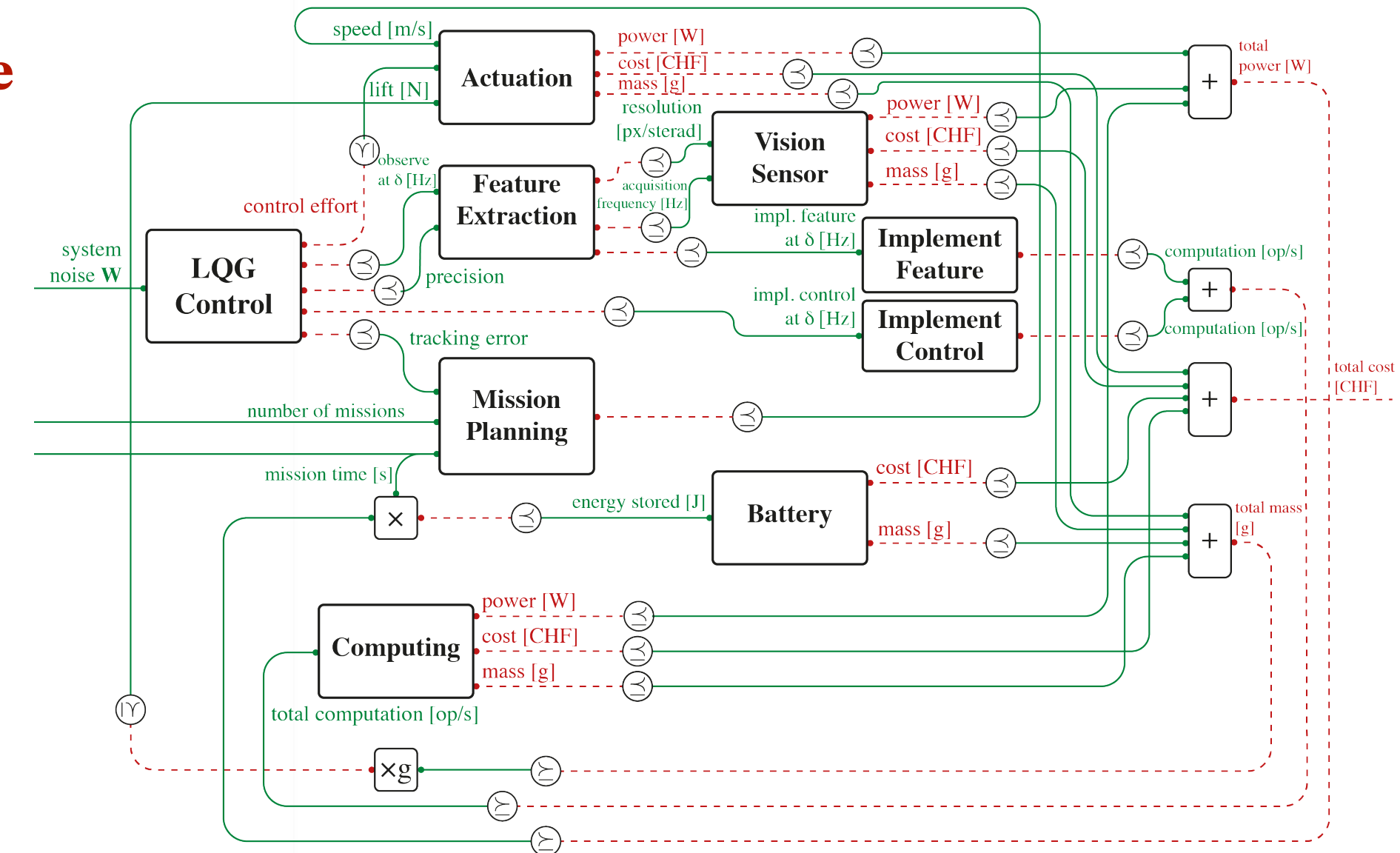
# Optimization semantics

► This is the semantics of **FixFunMinReq** as a family of optimization problems.

chosen by user



to minimize



variables

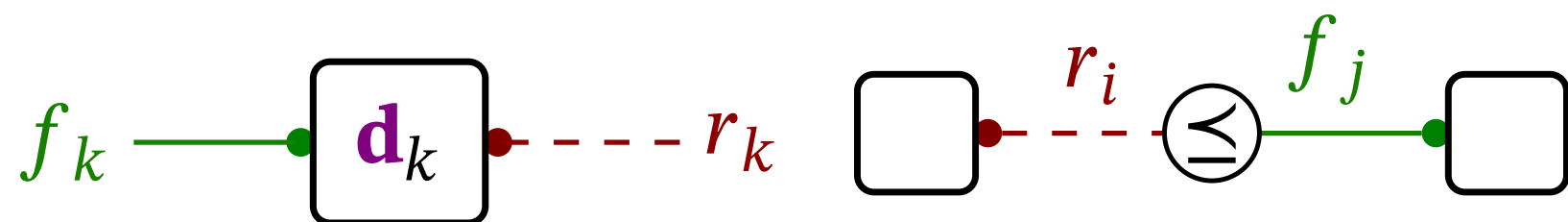
$$f_k \in \langle \mathbf{F}_k, \leq_{\mathbf{F}_k} \rangle$$

$$r_k \in \langle \mathbf{R}_k, \leq_{\mathbf{R}_k} \rangle$$

constraints

for each node:

for each edge:



$$d_k(f_k^*, r_k) = \top$$

$$r_i \leq f_j$$

component feasibility

co-design constraint

objective

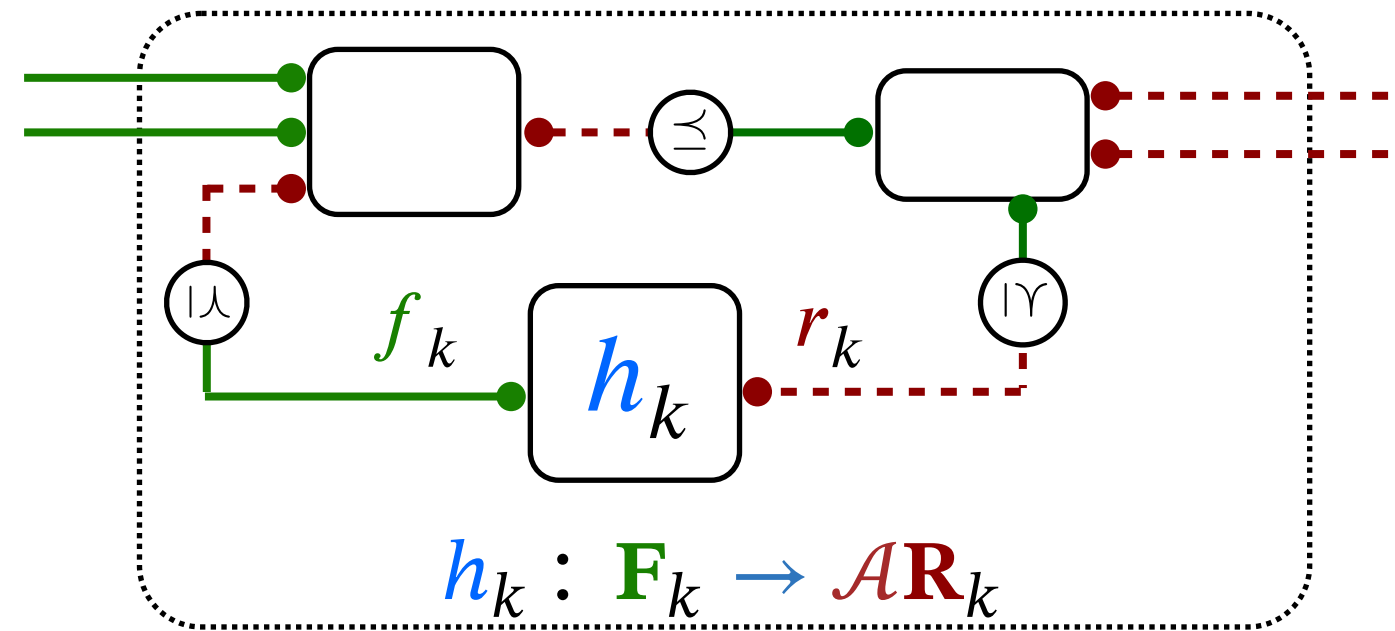
$$\text{Min } \bar{r}$$

- ! not convex
- ! not differentiable
- ! not continuous
- ! not even defined on continuous spaces



# Compositional solution of design problem queries

- Suppose that we are given the map  $h_k : \mathbf{F}_k \rightarrow \mathcal{AR}_k$  for all nodes in the co-design graph



✓ **Computationally tractable**

- Can we find the map  $h : \mathbf{F} \rightarrow \mathcal{AR}$  for the entire graph?

- Compositional approach:** just need to work out the composition formulas for all operations

$$\mathbf{solution}(\mathbf{composition}(a, b)) = \mathbf{composition}(\mathbf{solution}(a), \mathbf{solution}(b))$$

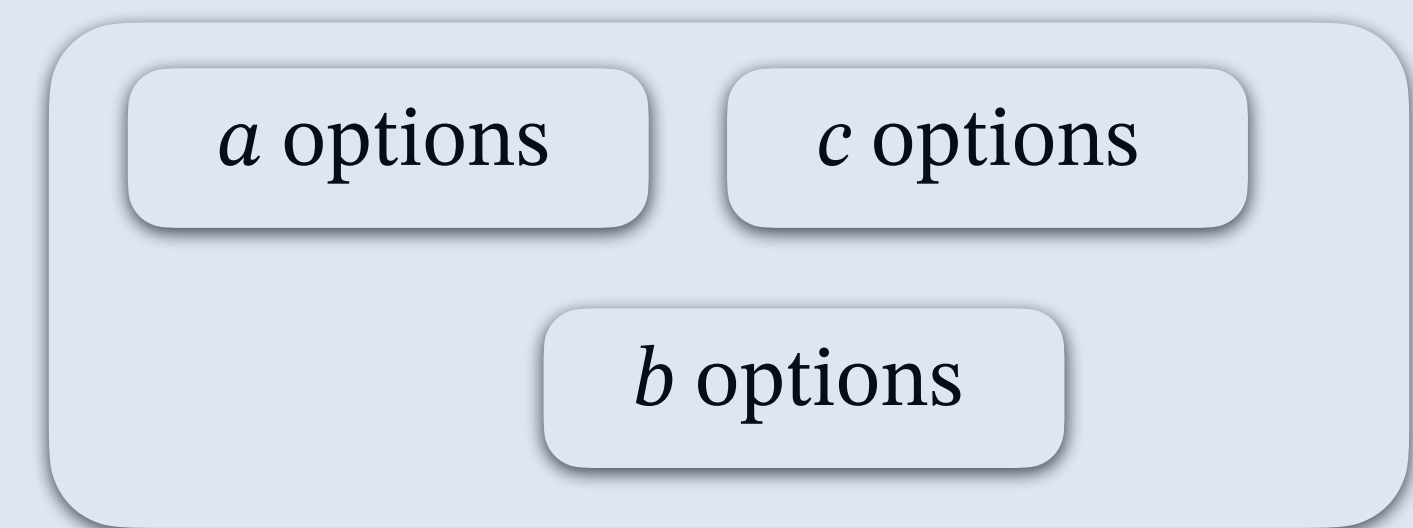
*... a functor between a category of problems and one of solutions*

- The set of **minimal** feasible **resources** can be obtained as the **least fixed point** of a monotone function in the space of anti-chain

- We have a **complete solution:** guaranteed to find the set of **all** optimal solutions (if empty, **certificate of infeasibility**)

- The complexity is **not combinatorial in the number of options** for each component

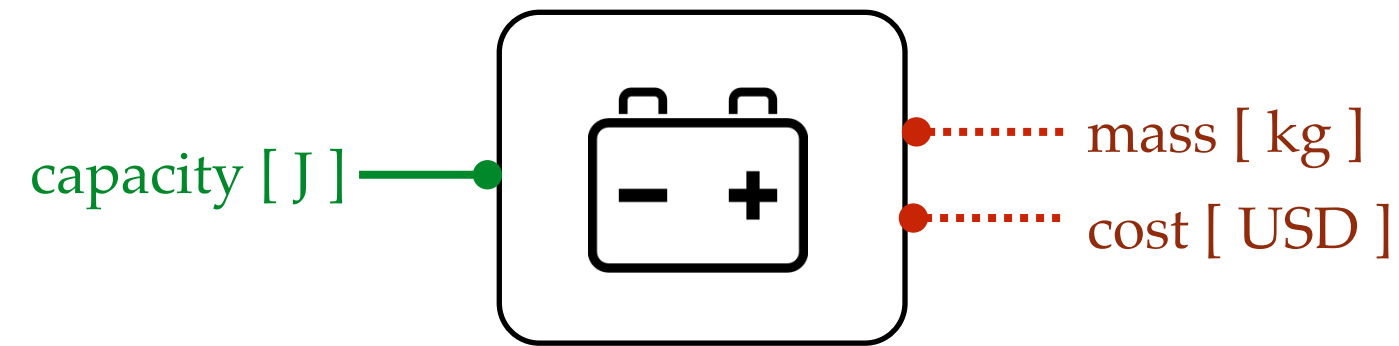
- The complexity depends on the **complexity of the interactions:** the co-design **constraints**



$$O(a + b + c)$$

# User-friendly interfaces

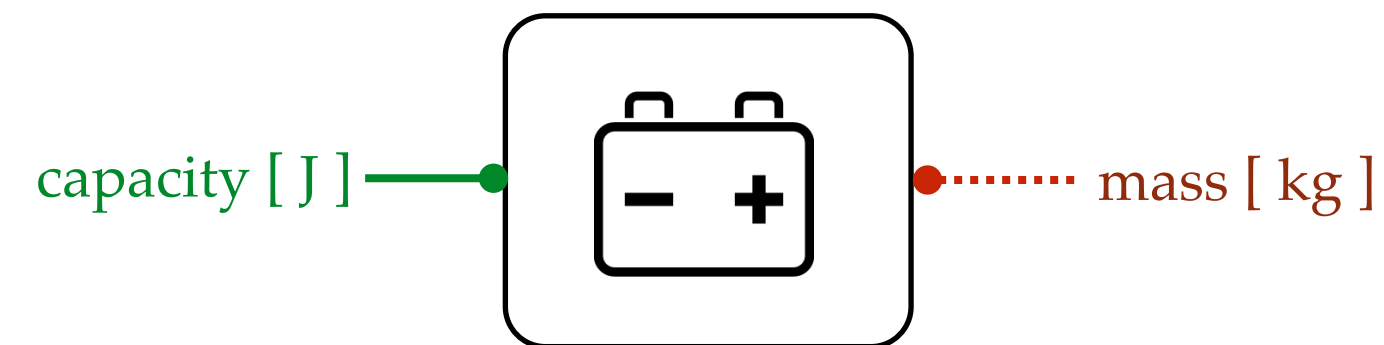
- ▶ “Catalogues”: already available designs



```
catalogue {  
  provides capacity [J]  
  requires mass [g]  
  requires cost [USD]  
  
  500 kWh ← mode11 → 100 g, 10 USD  
  600 kWh ← mode12 → 200 g, 200 USD  
  600 kWh ← mode13 → 250 g, 150 USD  
  700 kWh ← mode14 → 400 g, 400 USD  
}
```

... and a solver

- ▶ “First-principles”: analytical relations.



```
mcdp {  
  provides capacity [J]  
  requires mass [kg]  
  
  specific_energy_Li_Ion = 500 Wh / kg  
  
  required mass >= provided capacity / specific_energy_Li_Ion  
}
```



```
mcdp {  
  provides lift [N]  
  requires power [W]  
  c = 10.0 W/N2  
  required power ≥ c · provided lift2  
}
```

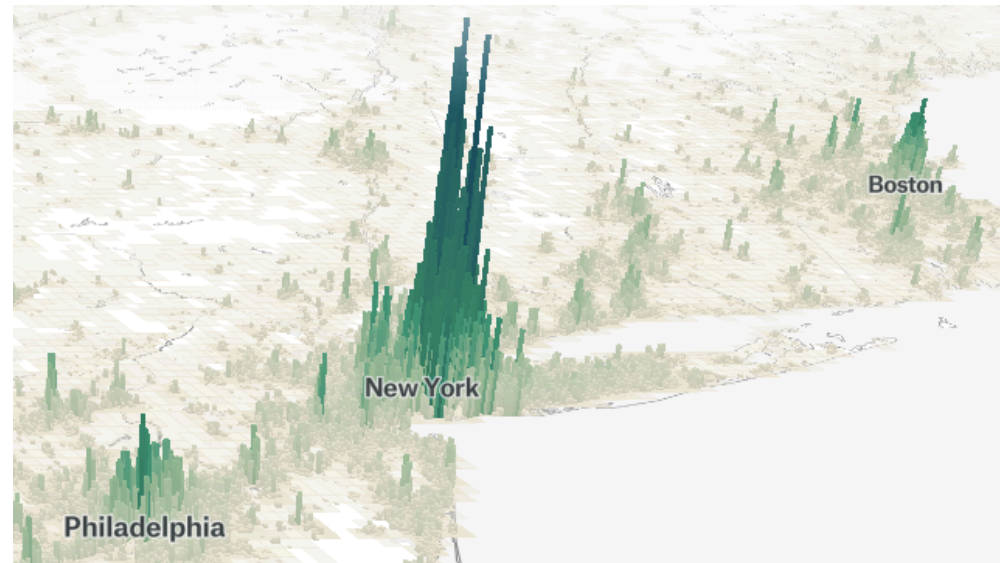


# Co-design across scales: from autonomy to mobility systems

- ▶ Mobility systems are **under pressure**

**Travel demand** is changing

*By 2050, 68% of population in cities*



Need for **service design** and **regulations**

*Over 1,000% ride-hailing increase in 2012-22*



Need to meet **sustainability goals**

*Cities cause 60% of GHGs, 30% from mobility*



- ▶ We look at the problem from the perspective of **municipalities** and **policy makers**

*How many vehicles should we allow?*

*Which infrastructure investments?*

*How performant?*

*Which services to encourage?*

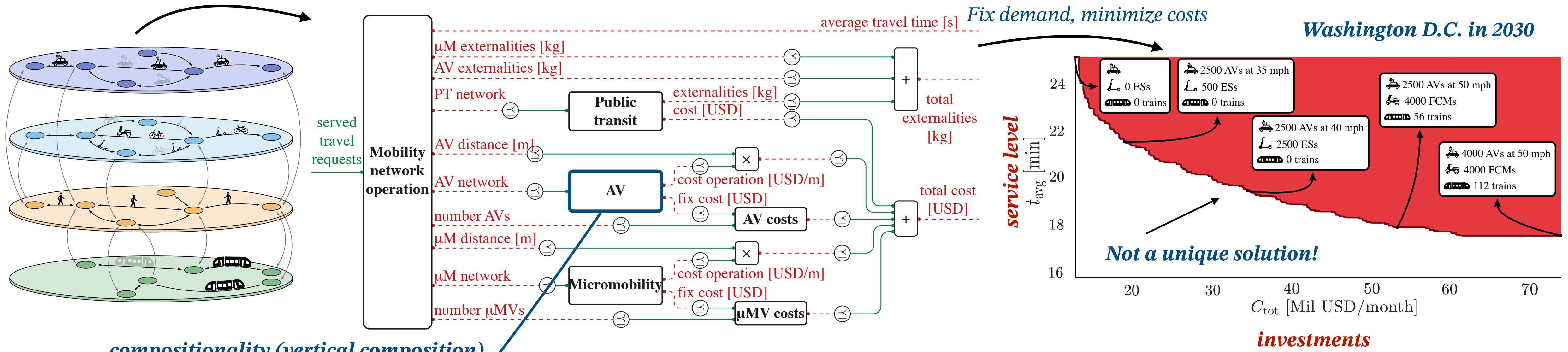
- ▶ Need for **demand-driven** co-design of **mobility solutions** and the **intermodal network** they enable

- ▶ Several **disciplines** involved (transportation science, autonomy, economics, policy-making)

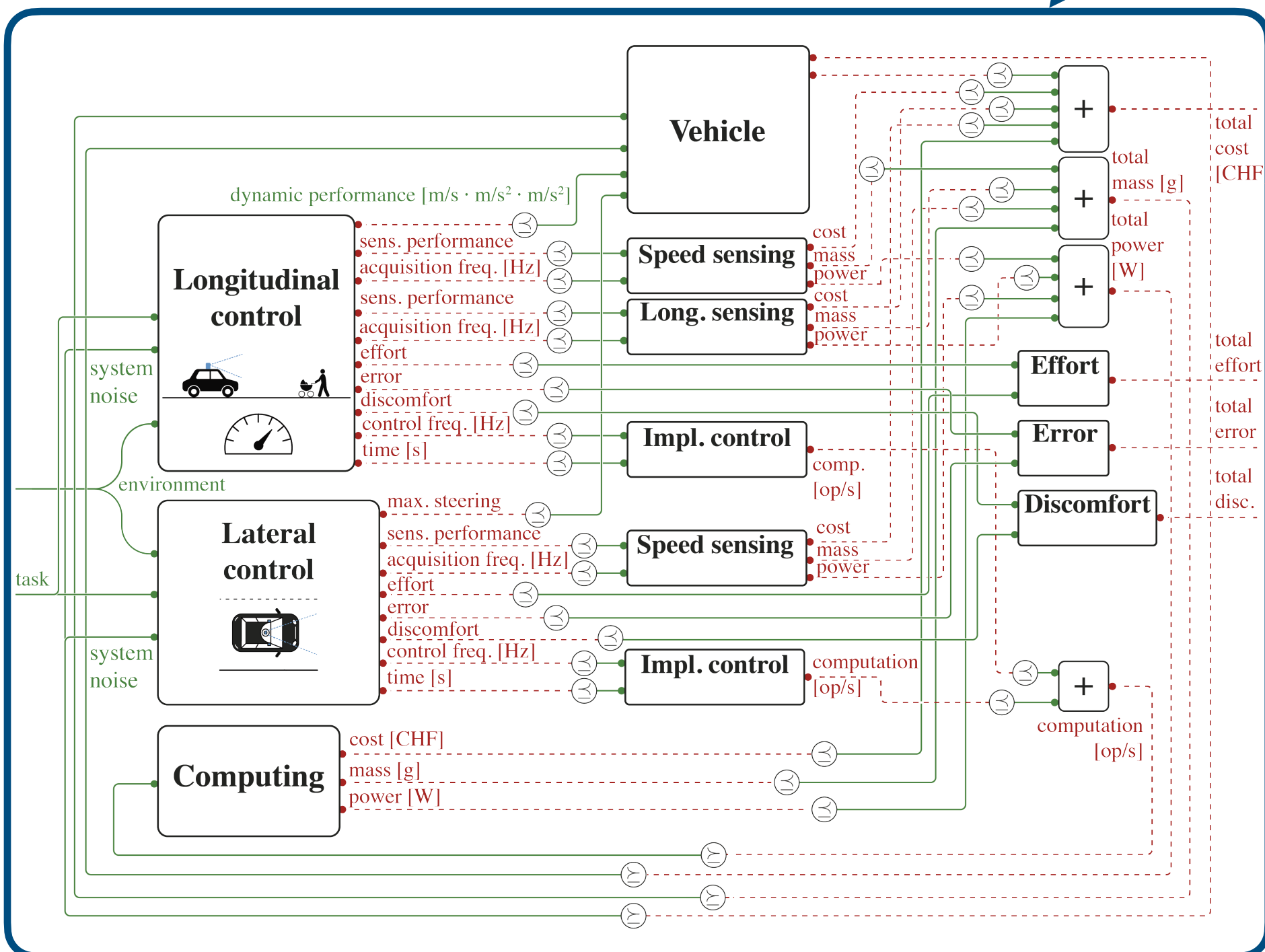




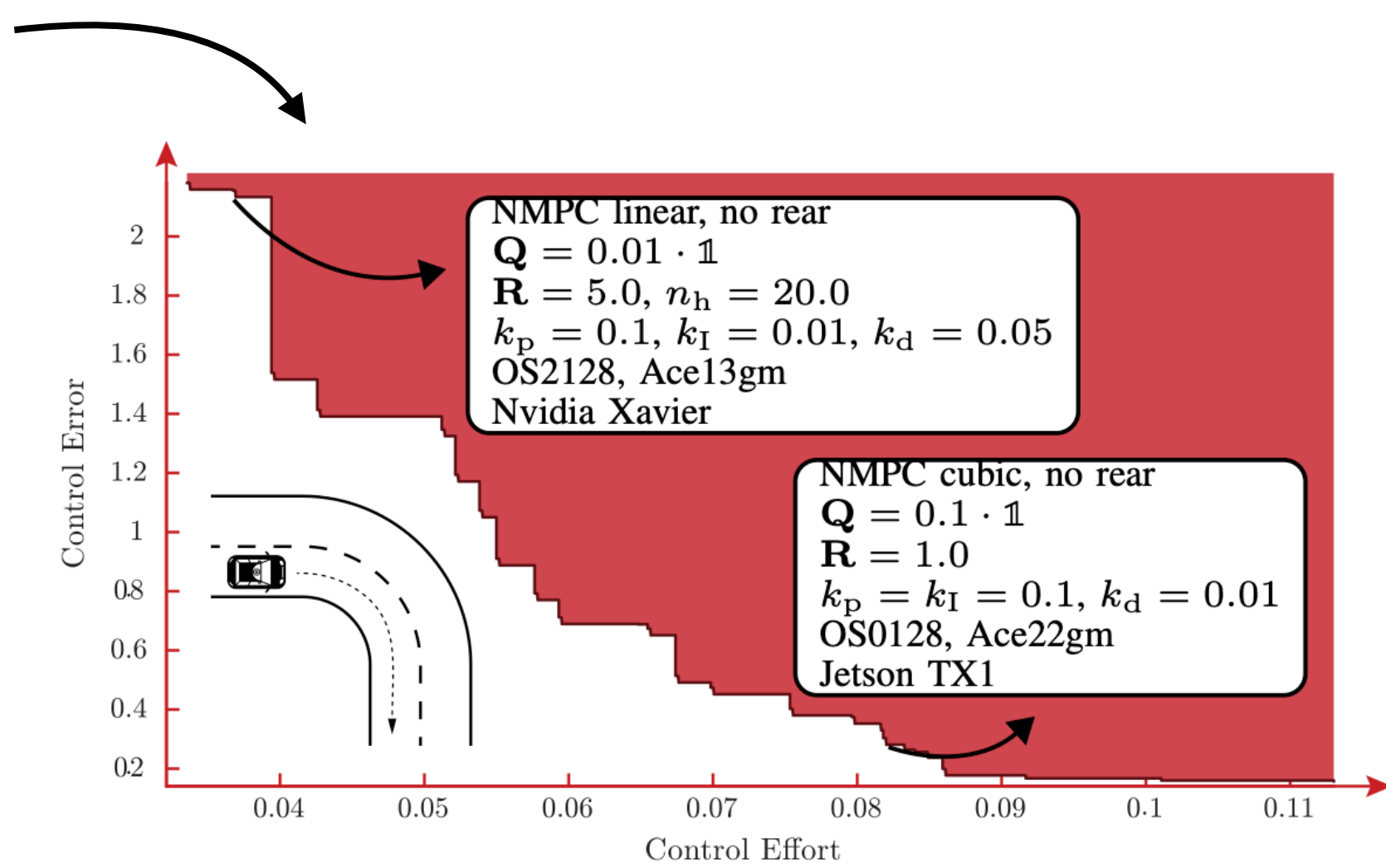
# Co-design to enable user-friendly tools to assess the impact of future mobility solutions



*compositionality (vertical composition)*



*Fix environment, task*



*Details about software and hardware implementations, in a way that was not possible before*



*Which solution is the best?*

**Continuous Collaborative Intellectually tractable**



# Agenda

## ► Motivation

- *New challenges of engineering design*
- *Motivation from autonomy and mobility*
- *Desiderata for co-design*

## ► Monotone Co-Design

- *Modeling design problems*
- *Examples across domains*
- *Design queries and optimization*
- *From autonomy to mobility systems*

## ► Strategic interactions

- *Game theory to deal with strategic interactions*
- *Hierarchical interactions in mobility systems*

## ► Outlook on future research

*Website containing all papers and more pointers:*

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## Complexity when designing complex systems

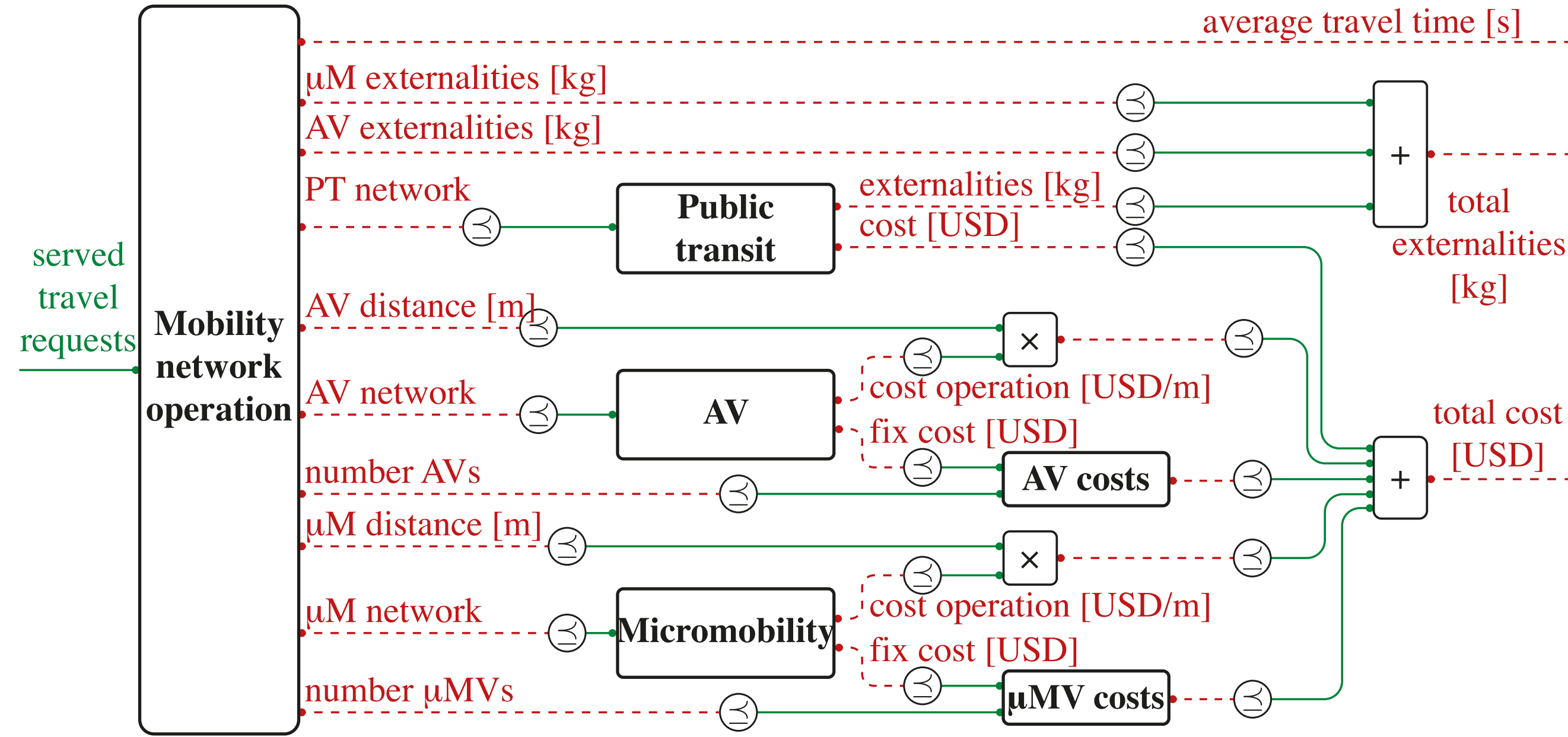
### Large systems

- Many components
- Heterogeneous natures
- Multiple objectives

### Strategic interactions

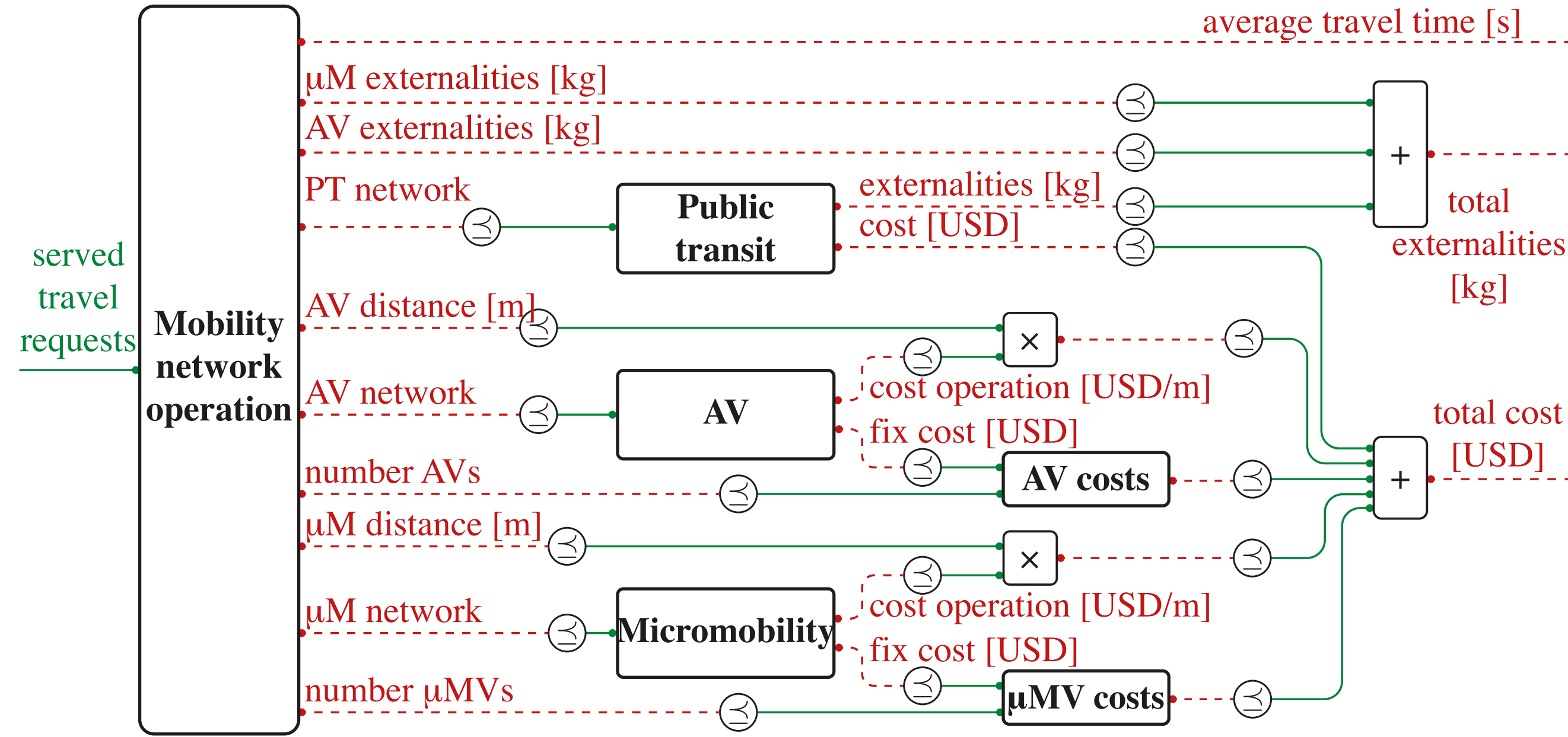
- Many agents
- Heterogeneous interactions
- Conflicts/collaborations

# Explicitly accounting for strategic interactions: towards co-design games





# Explicitly accounting for strategic interactions: towards co-design games



- ▶ Different **design problems** belong to different **stakeholders**
- ▶ Game theory: Multi-agent **strategic** decision making  
Allows one to **model interactions**
- ▶ The notion of **optimal designs** extends to **equilibria of designs**
- ▶ Towards a theory of **co-design games**

- ▶ Two **milestones** towards **co-design games**:

Co-design features **rich cost structures** (posets):

- “**Posetal Games**” (games with posetal preferences)

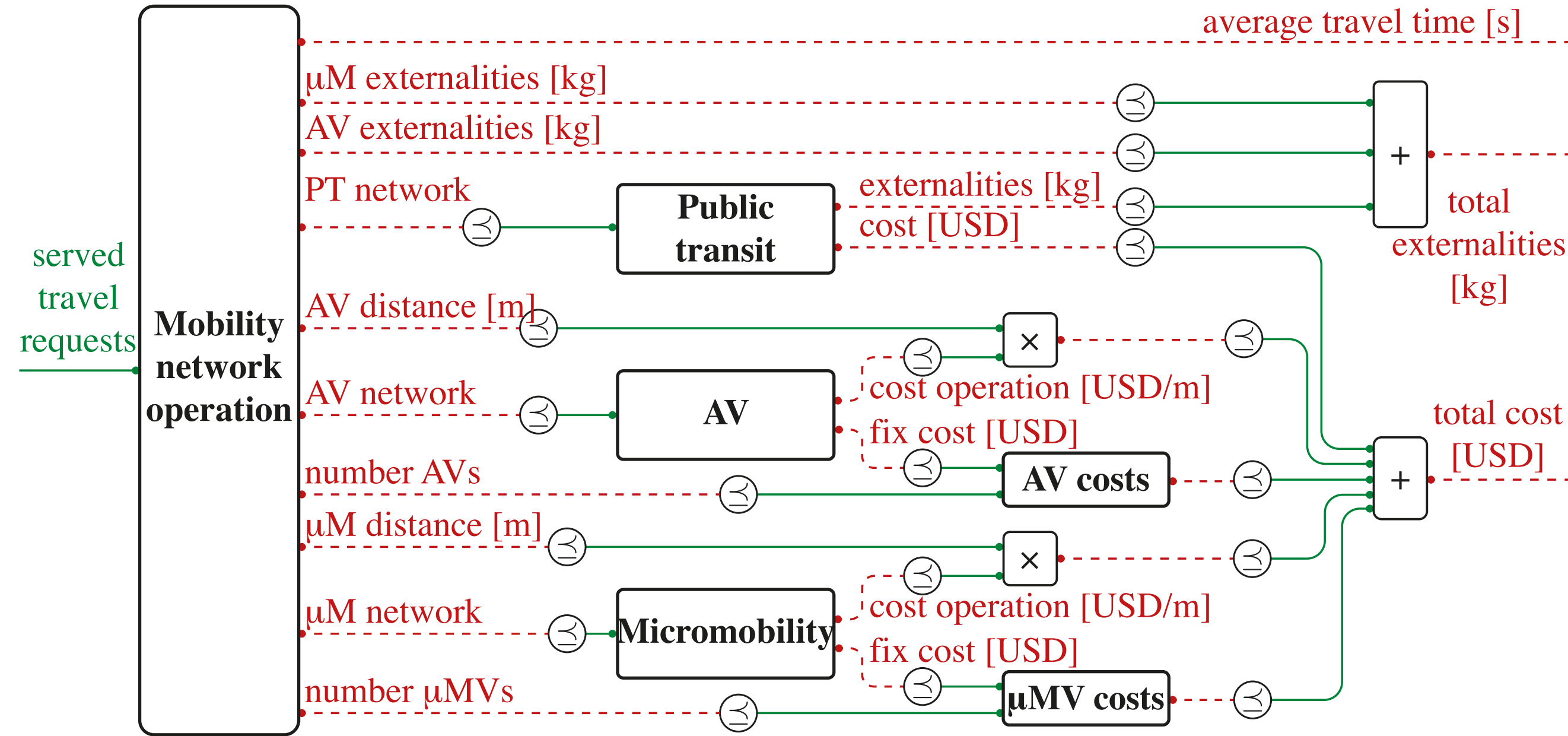
[RA-L’ 22]

**Interactions** are naturally **hierarchical**:

- **Mobility games** via Stackelberg

[ITSC’21 (*Best Paper Award*), ITSC’23]

# Explicitly accounting for strategic interactions: towards co-design games



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▶ Towards a theory of **co-design games**

▶ Two **milestones** towards **co-design games**:

Co-design features **rich cost structures** (posets): *Next time!*

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[RA-L’ 22]

**Interactions** are naturally **hierarchical**:

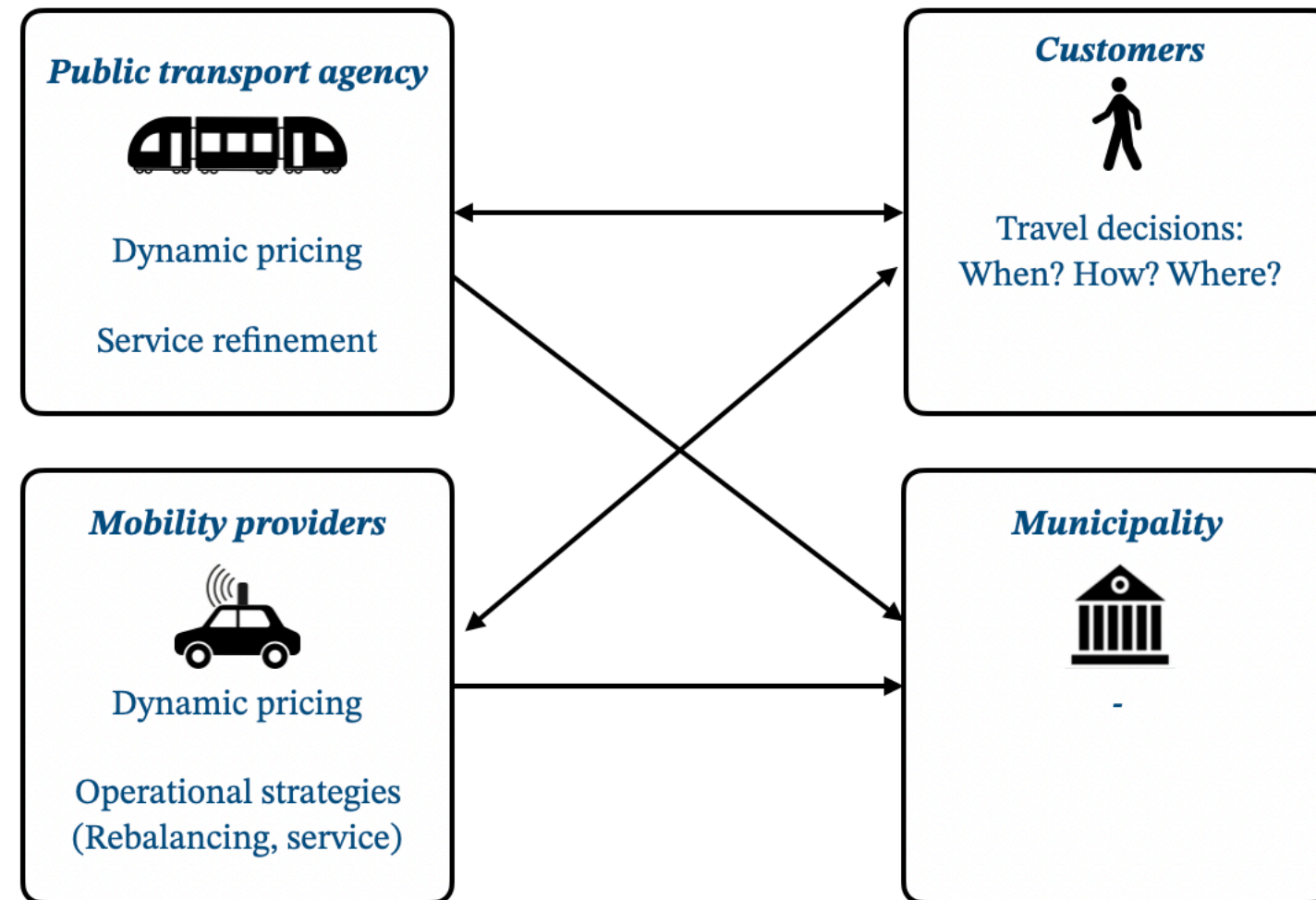
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[ITSC’21 (*Best Paper Award*), ITSC’23]

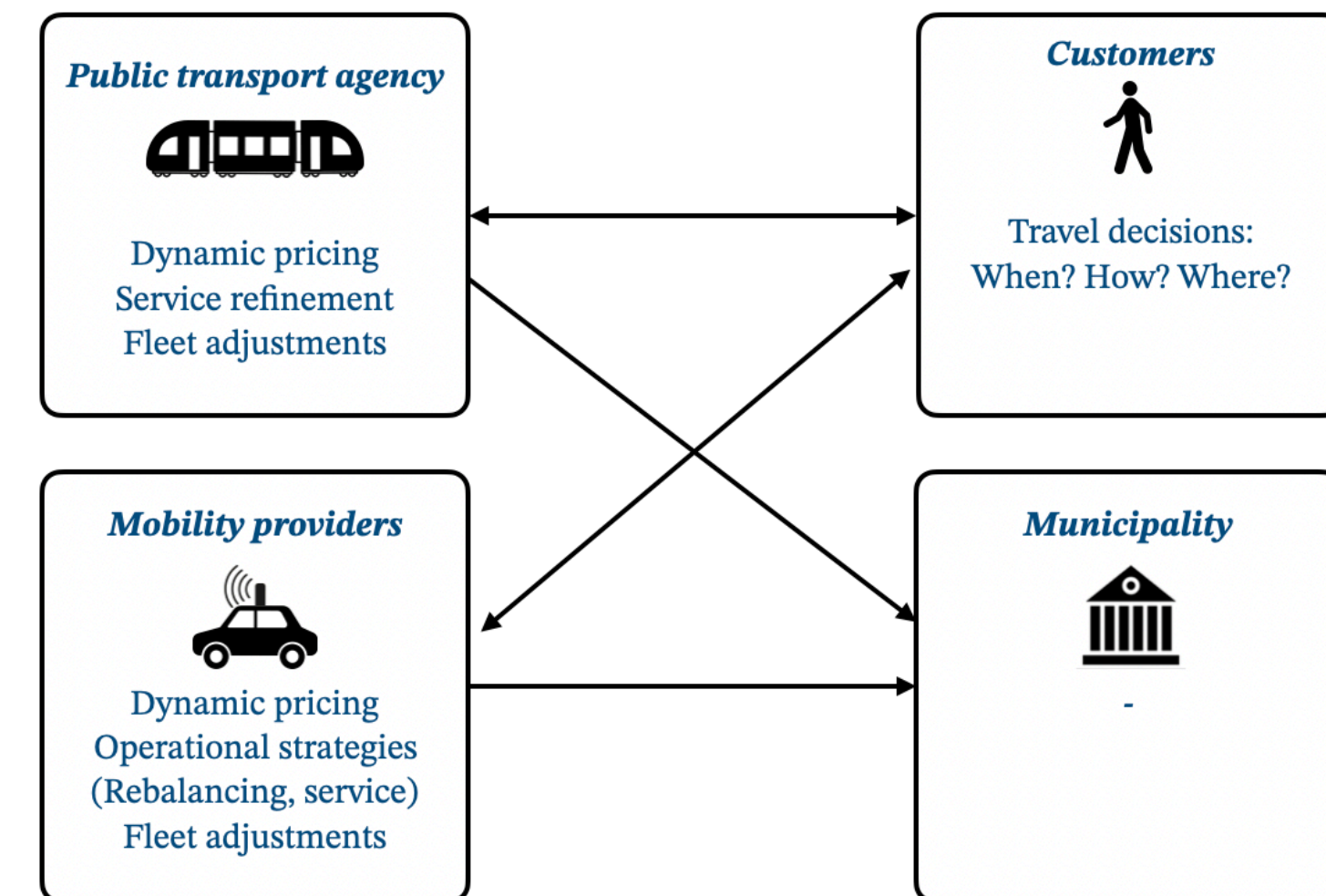


# Interactions between stakeholders are characterized by different time horizons

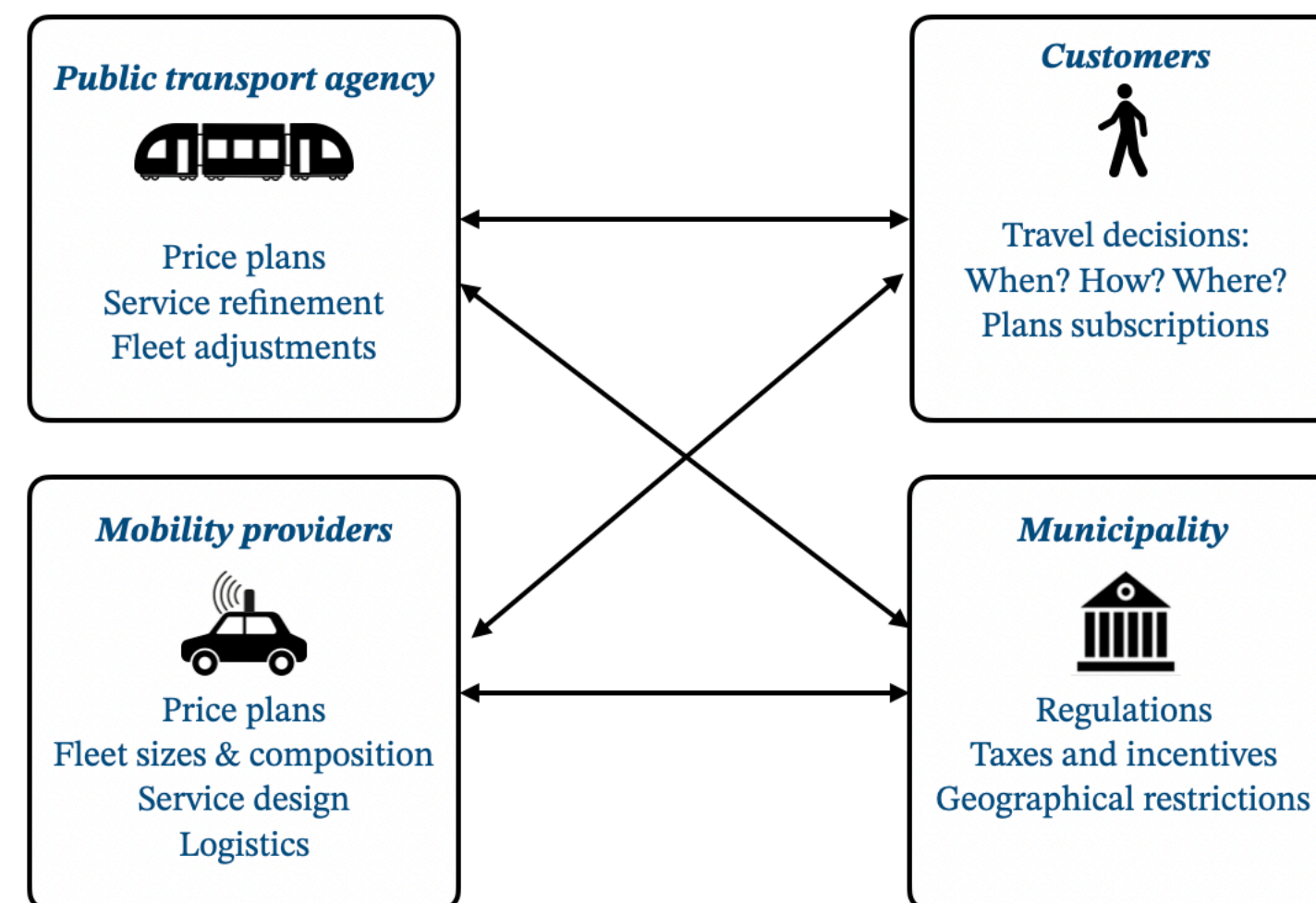
## Daily



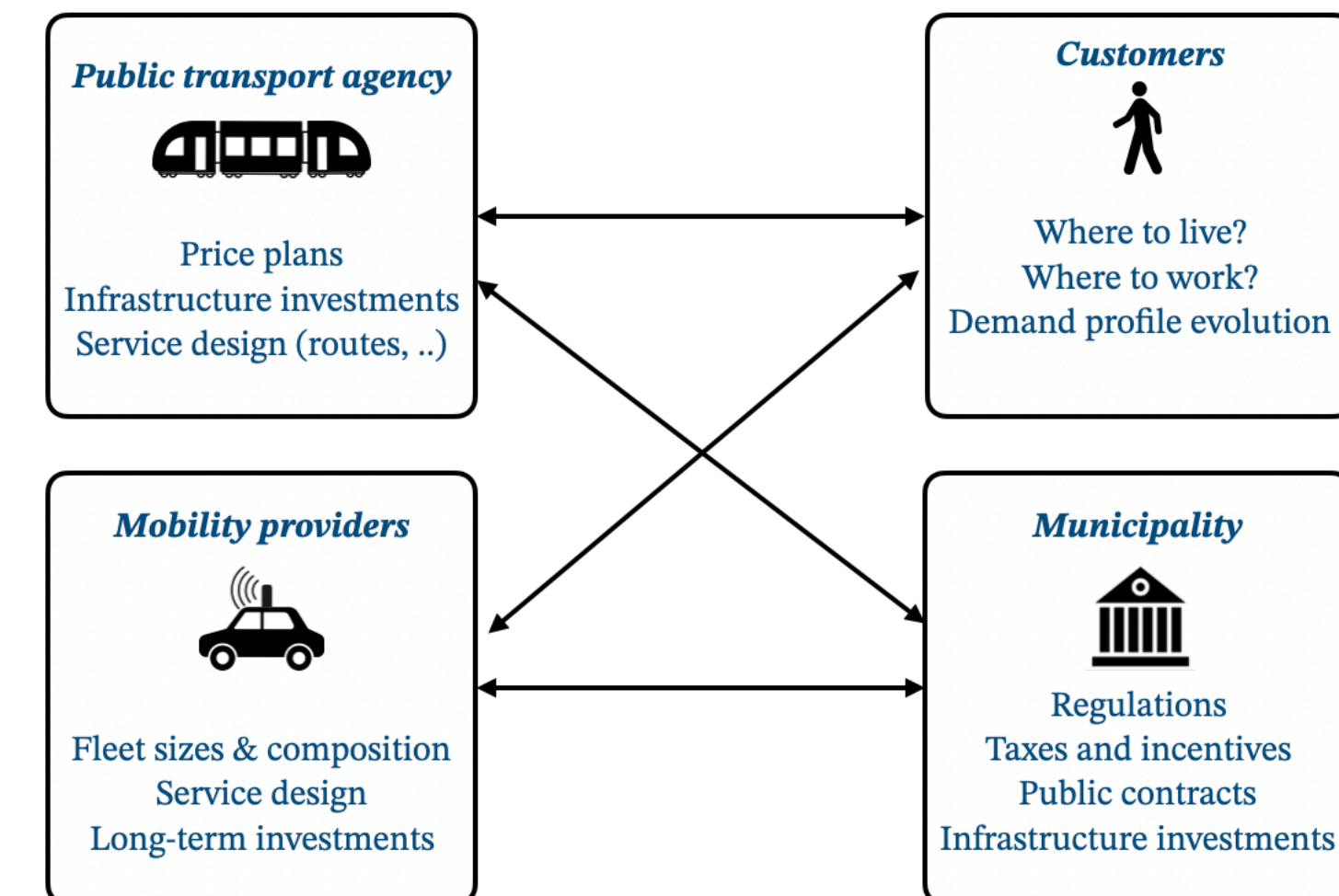
## Monthly



## Yearly

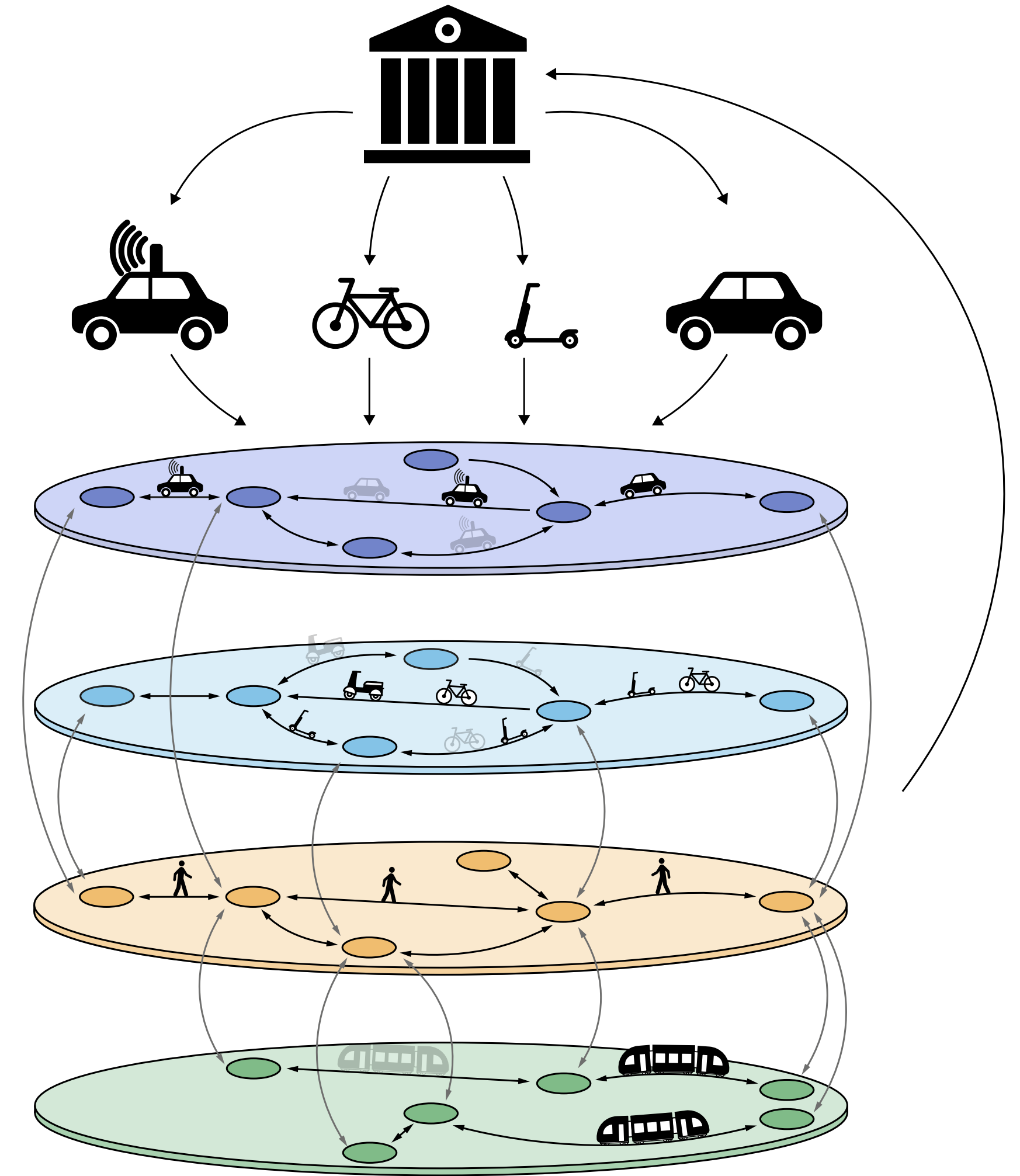


## Every five years



# Hierarchical nature of interactions can be modeled via Stackelberg games

- ▶ We model **sequential** interactions as a **game**:
  - **Municipality** plays **first**  
(choosing public transport **prices, taxes**)
  - The **mobility providers** interact **simultaneously** after the municipality (choosing **prices, fleet sizes**)
  - Customers **react** accordingly (choosing their **trip**)
- ▶ For instance:
  - **Municipalities** want to **minimize** *emissions*, and **maximize** *social welfare, performance*
  - **Mobility providers** want to **maximize** *return on investment*
- ▶ The payoff depends on a **low-level model** of the mobility system (e.g., a **simulator**, an **optimization problem**)
- ▶ We can compute equilibria via **backward induction**





# Considering strategic interactions for the city of Berlin, Germany

- ▶ We consider the city of **Berlin**, including:

## Municipality



### *Actions:*

- *Short-distance PT price*
- *Long-distance PT price*
- *Cutoff distance*
- *Distance-based tax for AVs*
- *Distance-based tax for empty AVs*

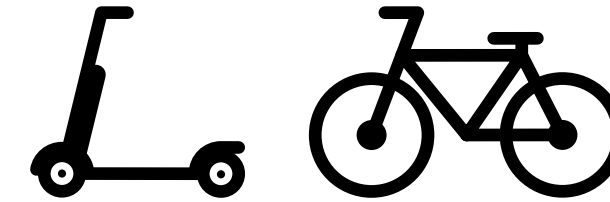
## AMoD operator



### *Actions:*

- *Propulsion*
- *Automation level*
- *Fleet size*

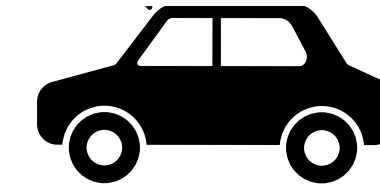
## Micro-mobility operator



### *Actions:*

- *Base price*
- *Mileage-dependent price*
- *Vehicle type*

## Taxi company

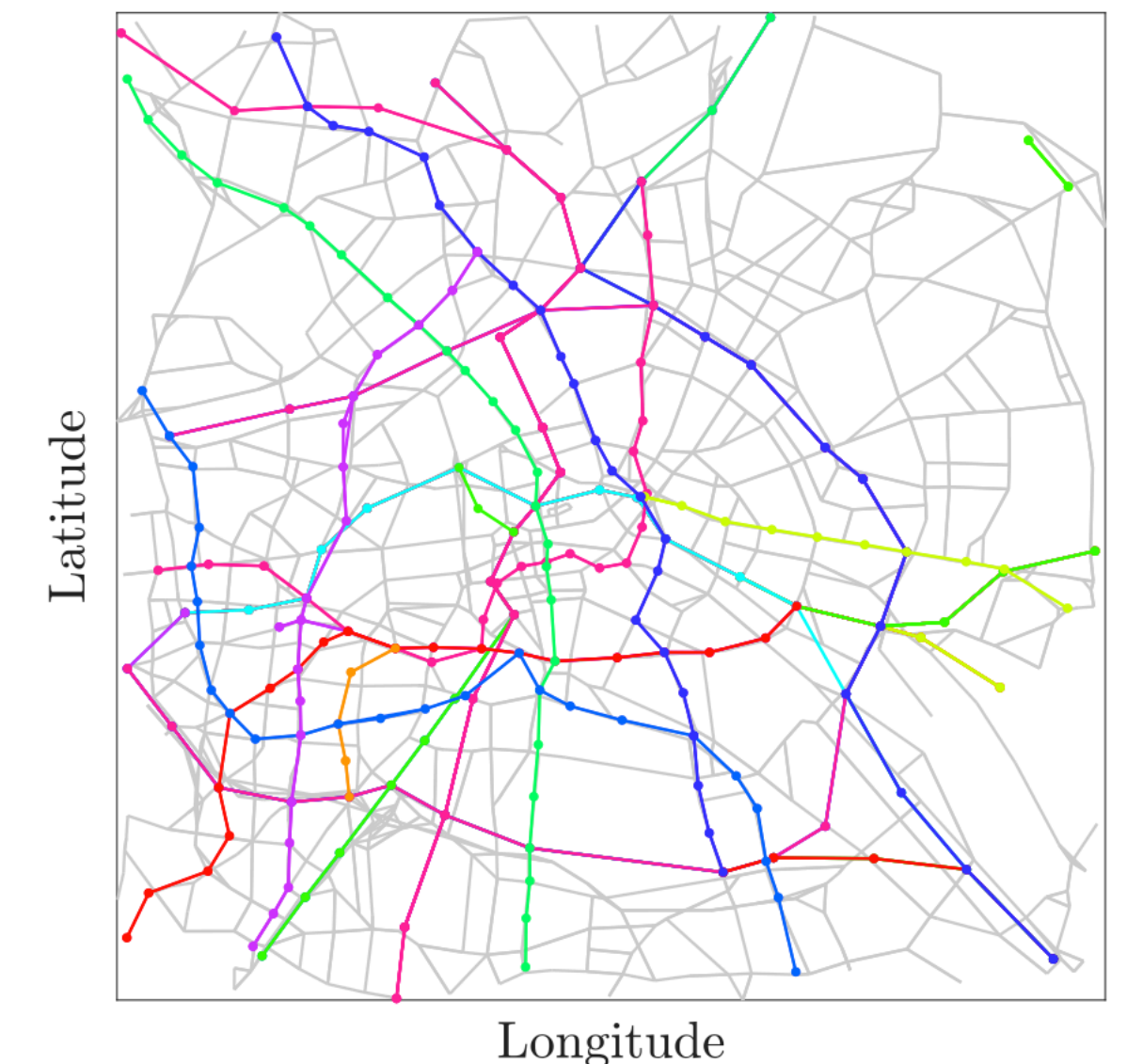


### *Actions:*

- *Base price*
- *Mileage-dependent price*

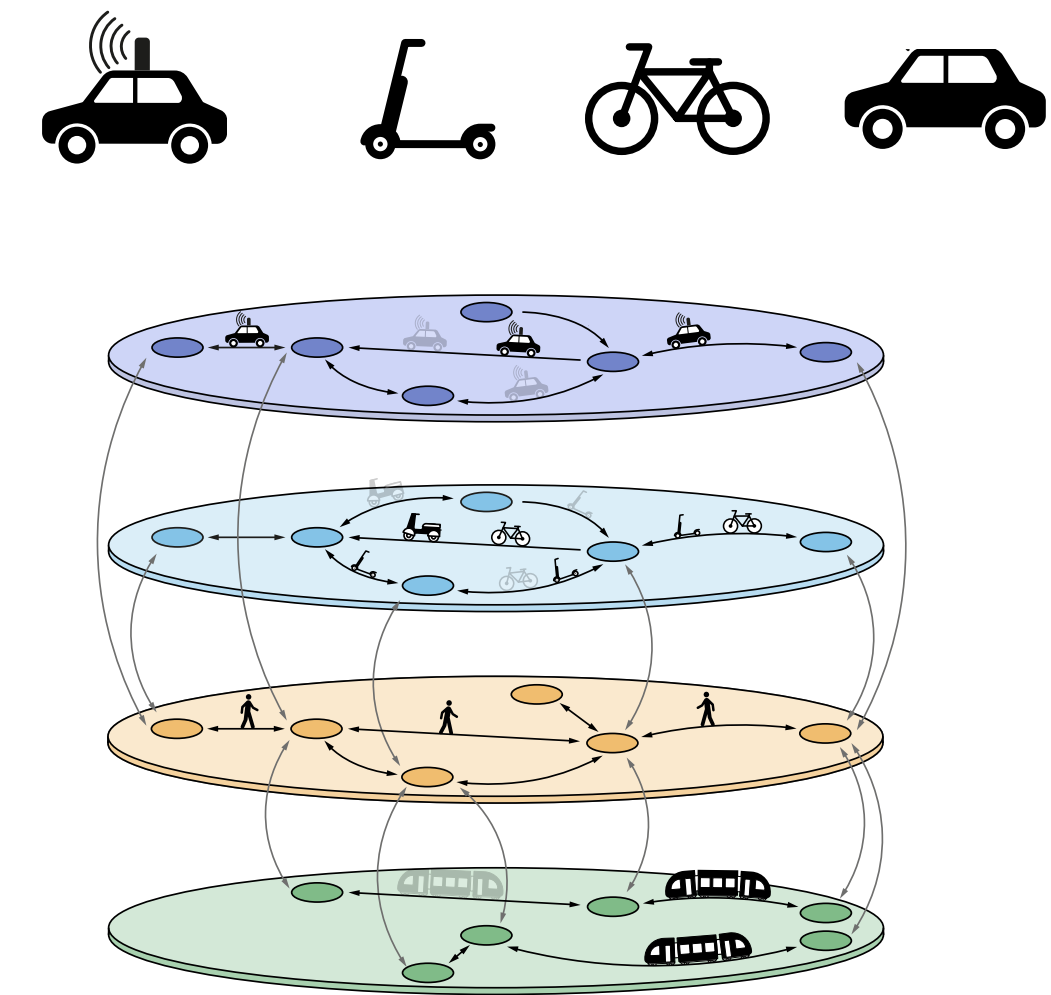
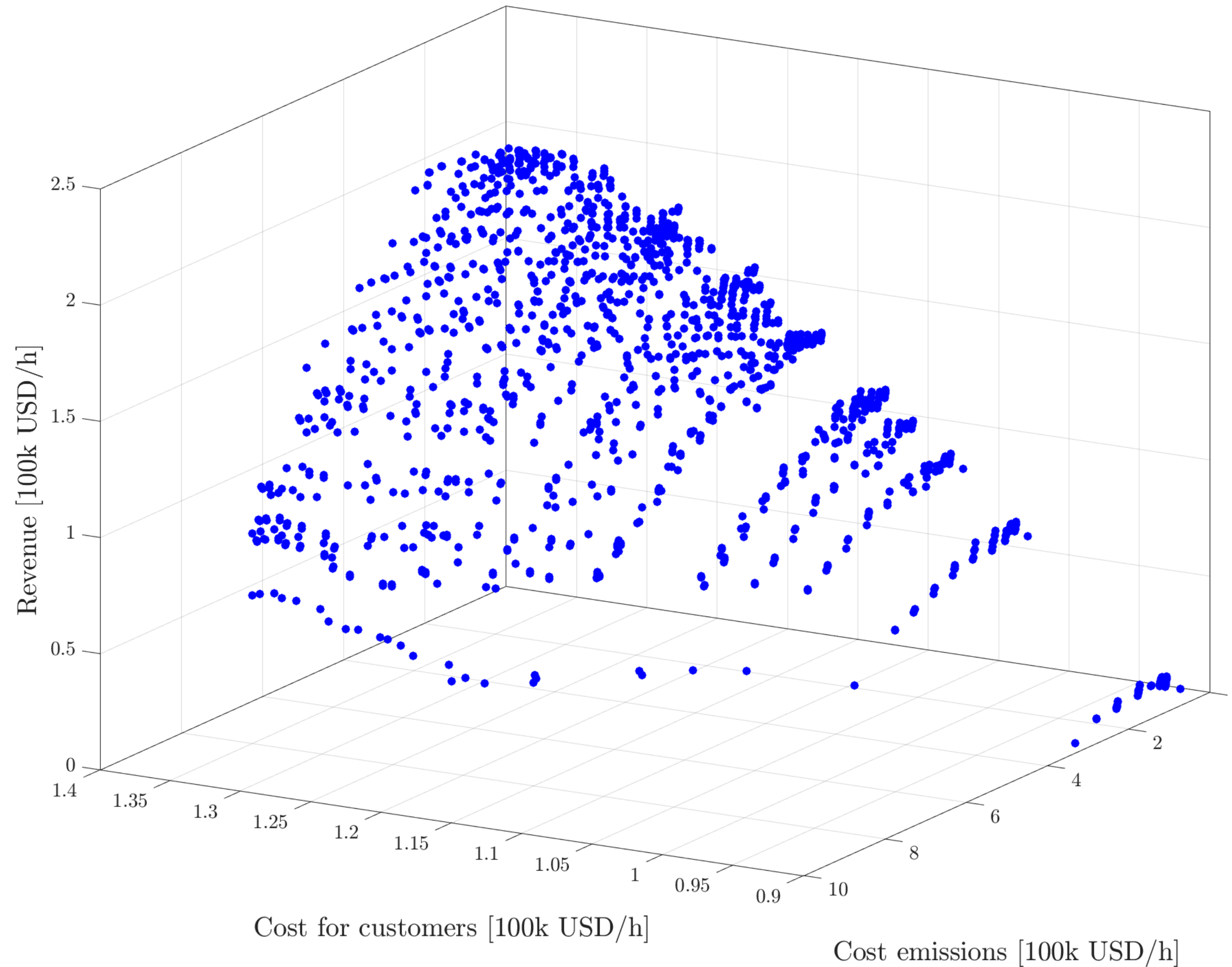
- ▶ Customers choose options by **minimizing** their **cost** (including **fare** and **value of time**)
- ▶ We consider 129,560 **real travel requests** and account for **congestion effects**
- ▶ We derive **vehicle-related parameters** and **costs** from **catalogues** and **official reports**

*Intermodal network in Berlin*



# Looking for equilibria of the simultaneous game between MSPs

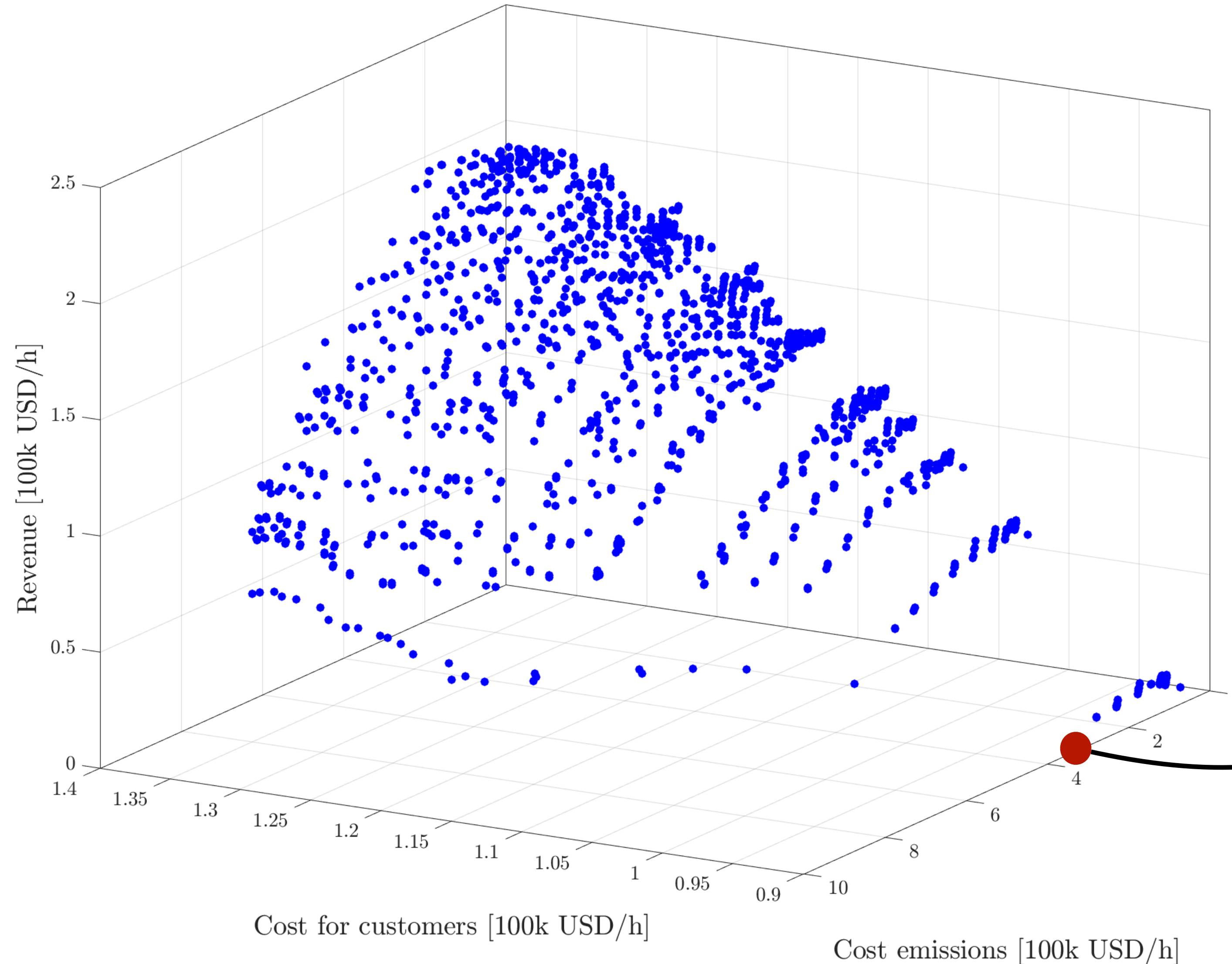
► First, we compute **equilibria** of the **simultaneous** game between mobility providers:





# Looking for equilibria of the sequential game

- ▶ We then compute the **equilibria** of the **sequential game**
- ▶ The **objective** of the **municipality** is pure *political* matter. For each choice, we produce **actionable information**:



## Customers-oriented City

### AMoD:

5,000 AVs, ICEV

### Micromobility:

E-scooters, with fares:

- Base: 1.20 USD
- Variable 1.21 USD/mile

### Municipality:

Public transit fares:

- SDP: 0 USD
- LDP: 0 USD

Taxes:

- 0 USD/mile both on full and empty vehicles

# Looking for equilibria of the sequential game

- ▶ We then compute the **equilibria** of the **sequential game**
- ▶ The **objective** of the **municipality** is pure *political* matter. For each choice, we produce **actionable information**:

## Revenue-oriented City

### AMoD:

5,000 AVs, ICEV

### Micromobility:

E-scooters, with fares:

- Base: 1.20 USD
- Variable 0.96 USD/mile

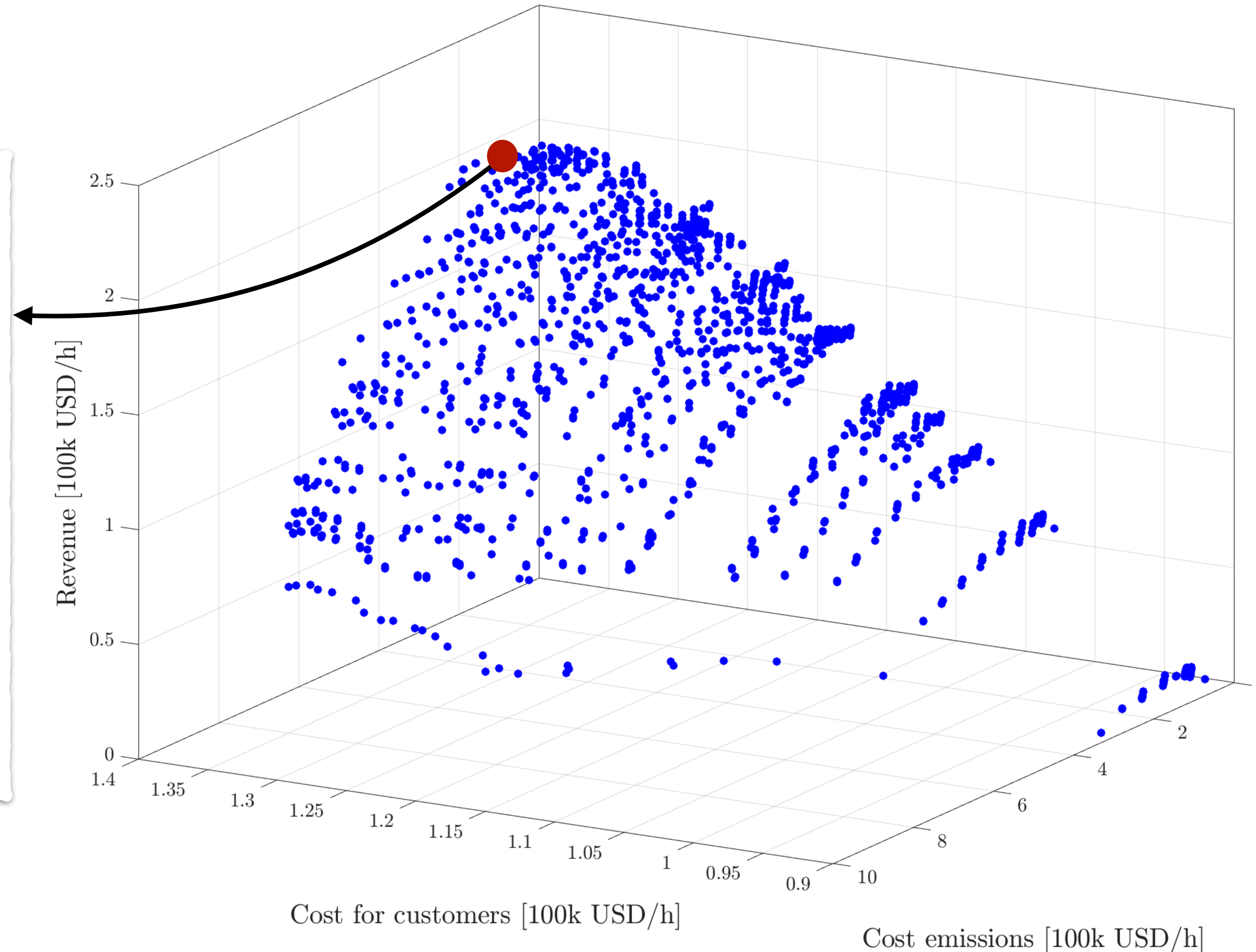
### Municipality:

Public transit fares:

- SDP: 3 USD
- LDP: 5 USD
- Cutoff: 1.55 miles

### Taxes:

- 1.28 USD/mile both on full and empty vehicles





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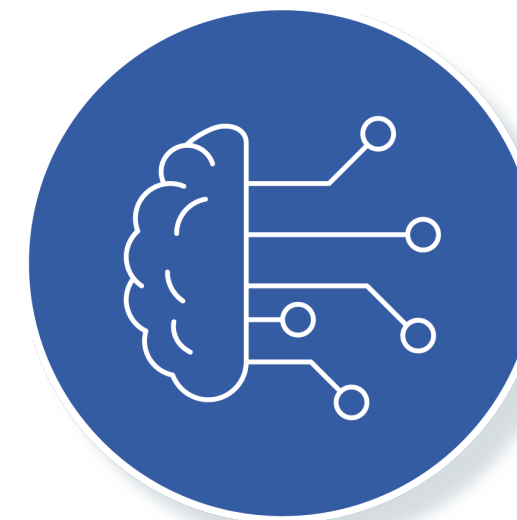
## ► Strategic interactions

- *Game theory to deal with strategic interactions*
- *Partial order games*

## ► Outlook on future research

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*Modeling & Algorithmic  
Foundations*



*Societal Applications*



*User-friendly Tools*

# My lab will be building the next generation tools for systems design optimization



*Modeling and Algorithmic  
Foundations*

Leveraging **optimization, control theory, game theory, domain theory, and applied category theory**:

- ▶ Extend and improve current **modeling & solution algorithms** for **multi-objective** design optimization
- ▶ Promote **interdisciplinarity** by bridging the gap between **standard optimization** and **co-design**
- ▶ Explicitly account for **strategic interactions** of stakeholders, developing a theory of **co-design games**



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*Societal Applications*



Mobility, networks, infrastructure  
Strategic interactions at all levels



Mission-driven autonomy



Aerospace, automotive, production chains, energy and data networks



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*Modeling and Algorithmic Foundations*



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Collaborative, intellectually tractable

*User-friendly Tools*



Authorities & Industry

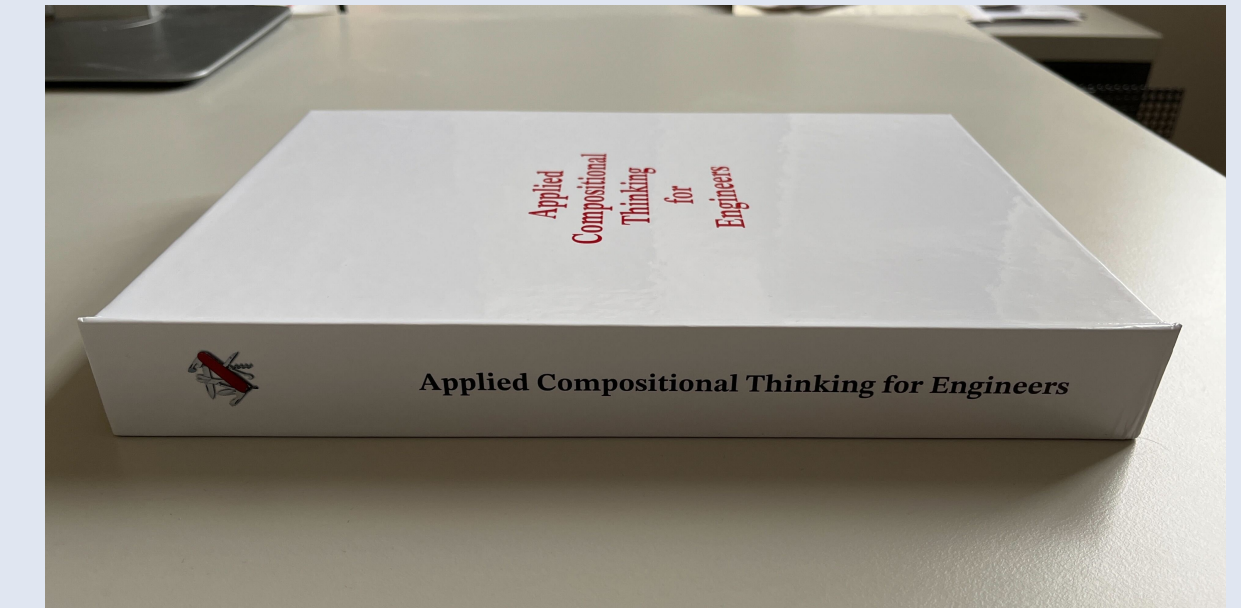


Literature, workshops, classes

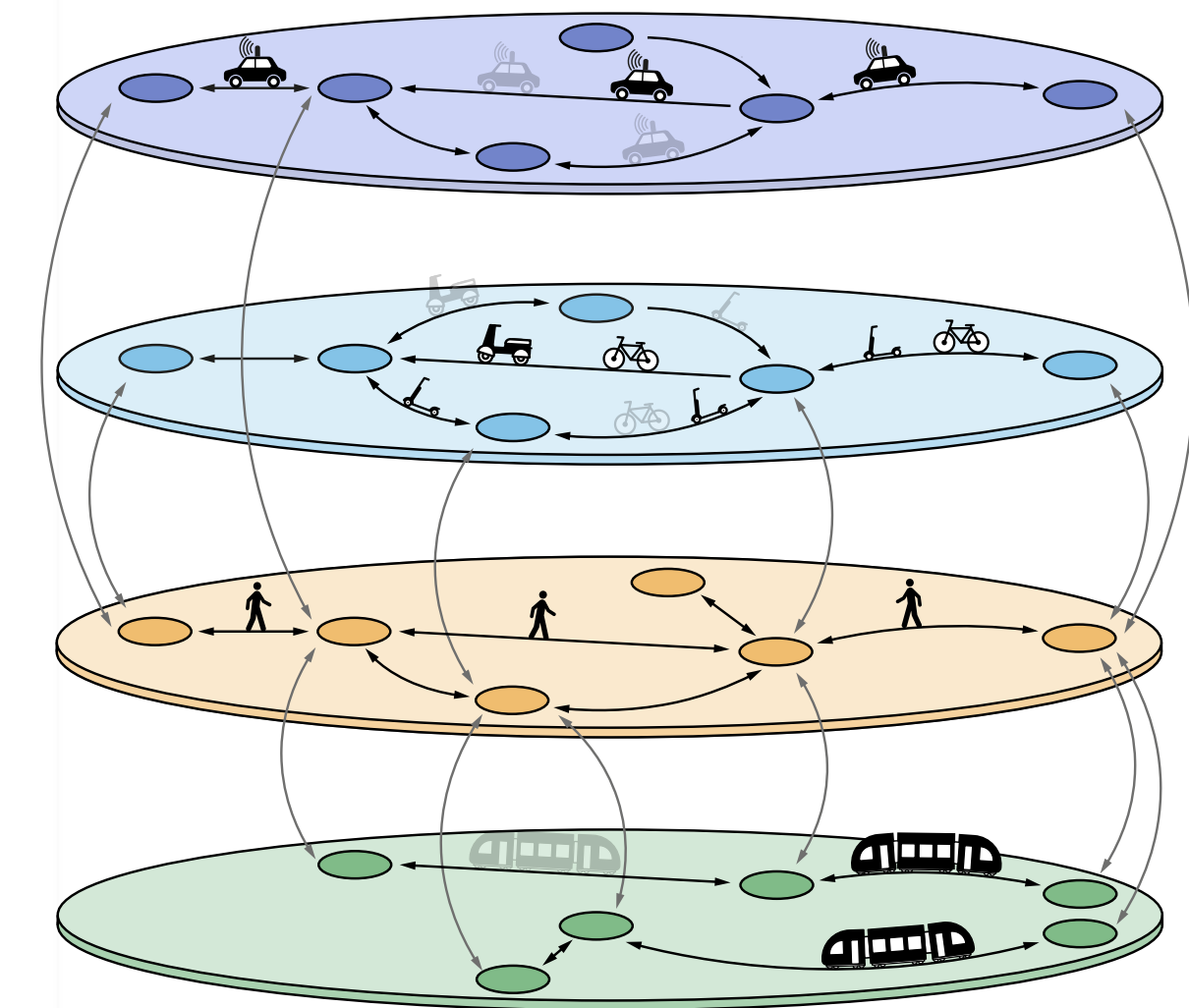
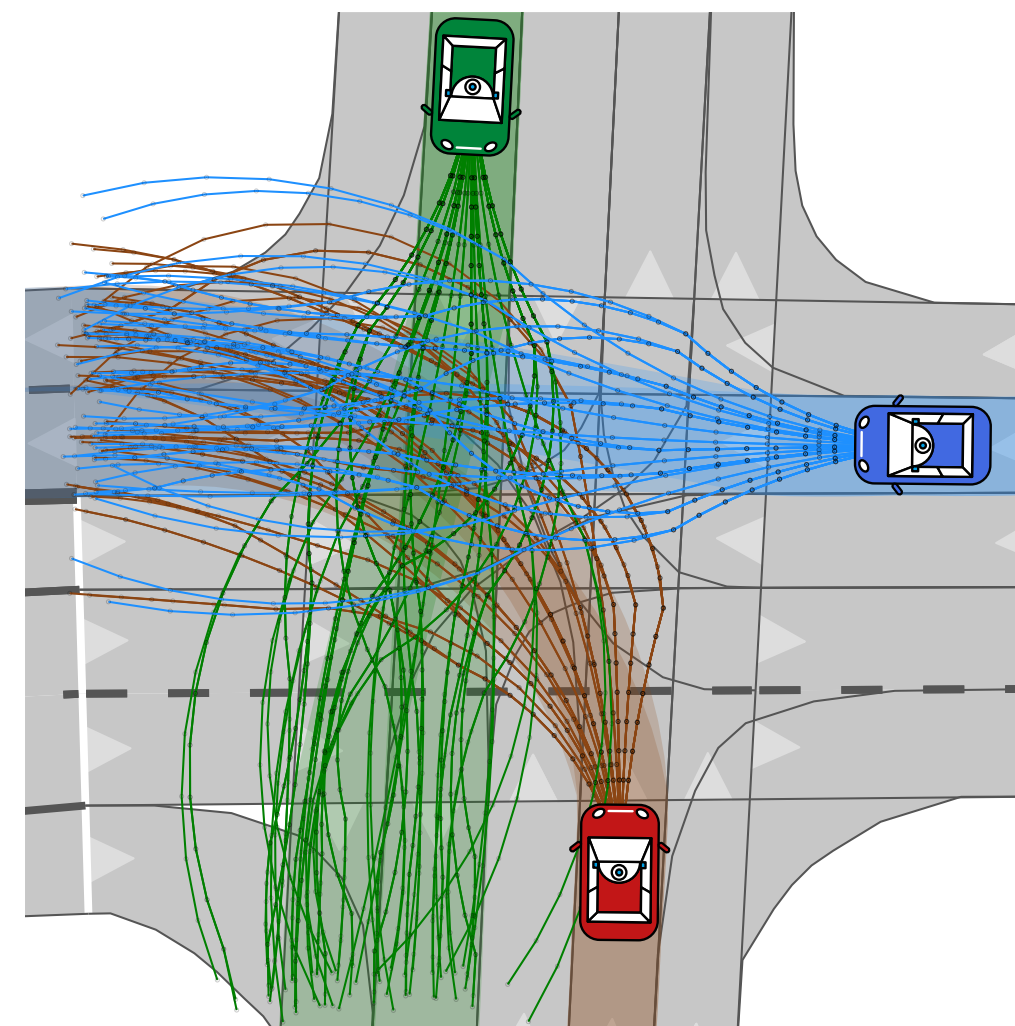
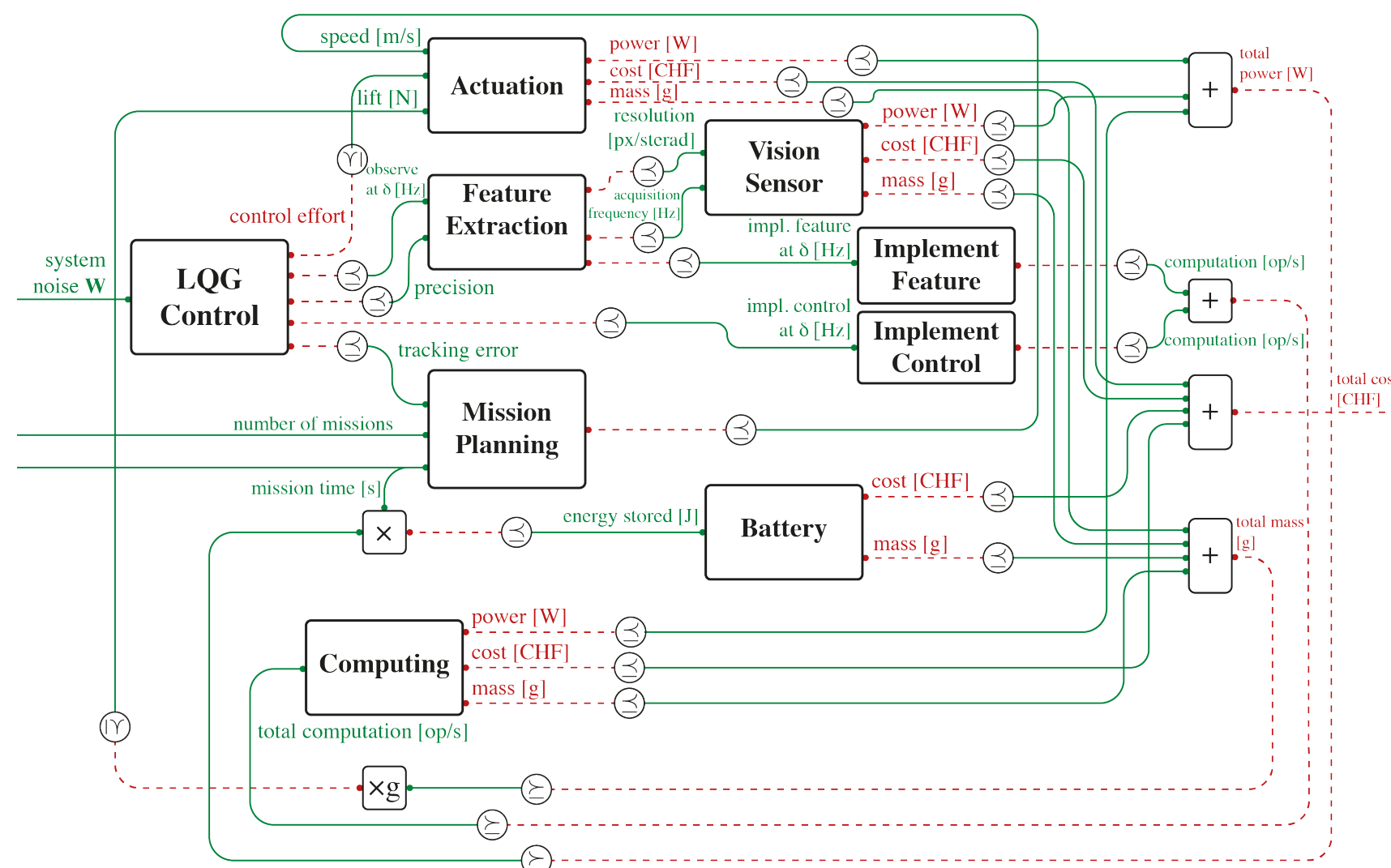


# Take-aways

- ▶ A new approach to **co-design** designed to work **across fields** and **scales**.
- ▶ It is:
  - **Compositional** horizontally and hierarchically.
  - Supports both **data-driven** and **model-based** components.
  - **Computationally tractable**.
  - **Intellectually tractable**.
- ▶ Future: extend **modeling** and **algorithmic** capabilities
- ▶ We need to account for **strategic interactions** of **designers**:
  - **Posetal games**: A new class of games, where **utilities** are **posets**
- ▶ Future: **uncertainty** and **computational** schemes



Access the book at:  
<https://bit.ly/3qQNrdR>



## Related references

- ▶ A. Censi, “A Mathematical Theory of Co-Design”, *arXiv preprint arXiv:1512.08055*, 2015.
- ▶ A. Censi, J. Lorand, G. Zardini, “Applied Compositional Thinking for Engineers”, *work-in-progress book*, 2023.
  
- ▶ G. Zardini, D. Milojevic, A. Censi, E. Frazzoli, “Co-Design of Embodied Intelligence: A Structured Approach”, *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2021.
- ▶ G. Zardini, A. Censi, E. Frazzoli, “Co-Design of Autonomous Systems: From Hardware Selection to Control Synthesis”, *EUCA European Control Conference (ECC)*, 2021.
- ▶ G. Zardini, Z. Suter, A. Censi, E. Frazzoli, “Task-driven Modular Co-Design of Vehicle Control Systems”, *IEEE Conference on Decision and Control (CDC)*, 2022.
- ▶ G. Zardini, N. Lanzetti, A. Censi, E. Frazzoli, M. Pavone, “Co-Design to Enable User-Friendly Tools to Assess the Impact of Future Mobility Solutions”, *IEEE Transactions on Network Science and Engineering*, 2023.
- ▶ G. Zardini, N. Lanzetti, M. Pavone, E. Frazzoli, “Analysis and Control of Autonomous Mobility-on-Demand Systems”, *Annual Review of Control, Robotics, and Autonomous Systems*, 2022.
  
- ▶ A. Zanardi\*, G. Zardini\*, S. Srinivasan, S. Bolognani, A. Censi, F. Dörfler, E. Frazzoli, “Posetal Games: Efficiency, existence, and refinement of equilibria in games with prioritized metrics”, *IEEE Robotics and Automation Letters*, 2022.
- ▶ G. Zardini, N. Lanzetti, L. Guerrini, S. Bolognani, E. Frazzoli, F. Dörfler, “Game Theory to Study Interactions Between Mobility Stakeholders”, *IEEE International Intelligent Transportation Systems Conference (ITSC)*, **Best Paper Award**, 2021.

*Co-Design  
basics*

*Co-Design  
of autonomy,  
mobility*

*Strategic  
Interactions*



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- ▶ Future: **uncertainty** and **computational** schemes
- ▶ **Collaborators** for the presented works

Zelio Suter

Laura Guerrini

Dejan Milojevic

Nicolas Lanzetti

Alessandro Zanardi

Dr. Jonathan Lorand

Dr. Saverio Bolognani

Dr. Andrea Censi

Prof. Florian Dörfler

Prof. Marco Pavone

Prof. Emilio Frazzoli

**ETH** zürich

**Stanford**  
University

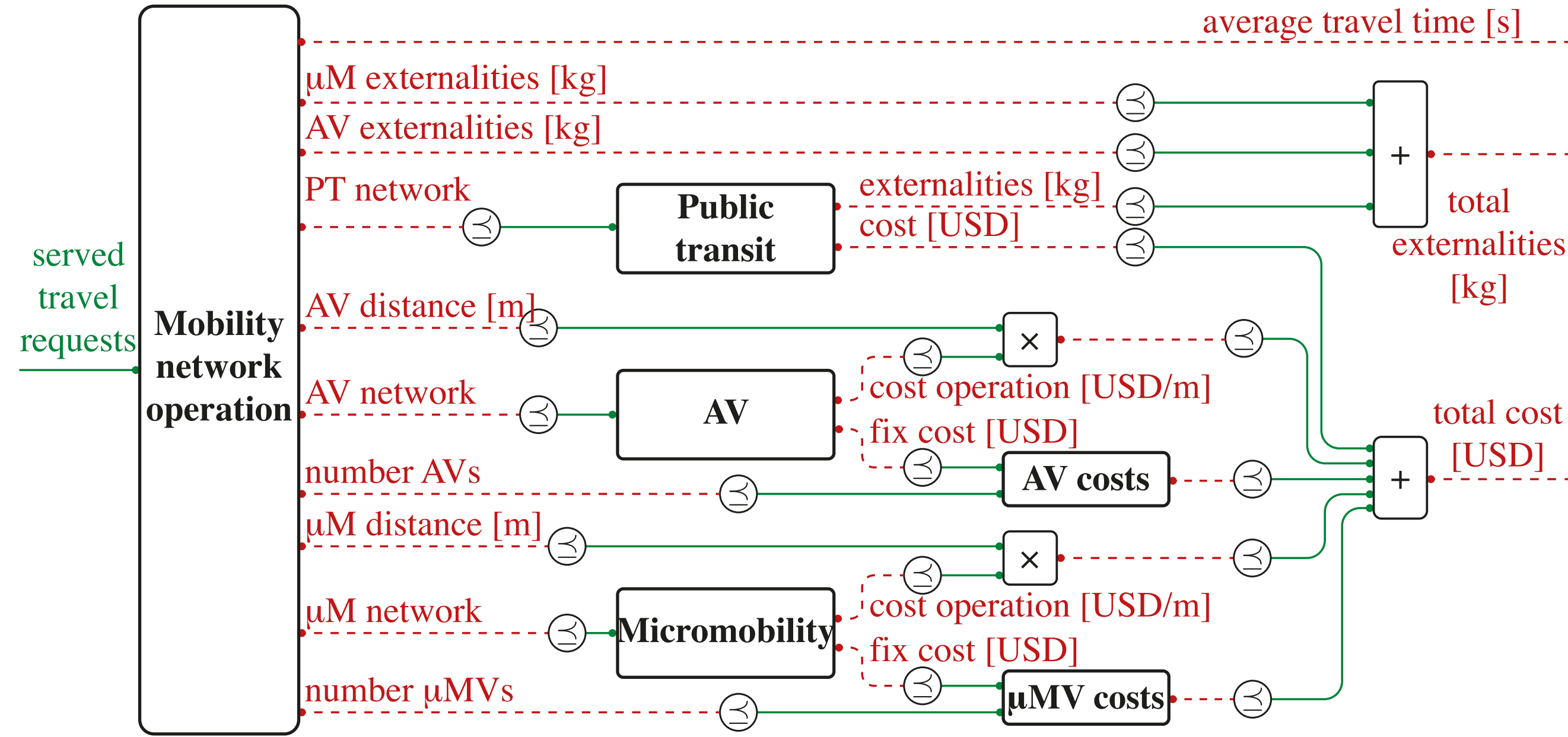
*Questions?*



**I'm hiring!**



# Explicitly accounting for strategic interactions: towards co-design games



- ▶ Different **design problems** belong to different **stakeholders**
- ▶ Game theory: Multi-agent **strategic** decision making  
Allows one to **model interactions**
- ▶ The notion of **optimal designs** extends to **equilibria of designs**
- ▶ Towards a theory of **co-design games**

- ▶ Two **milestones** towards **co-design games**:

Co-design features **rich cost structures** (posets):

- “**Posetal Games**” (games with posetal preferences)

[RA-L’ 22]

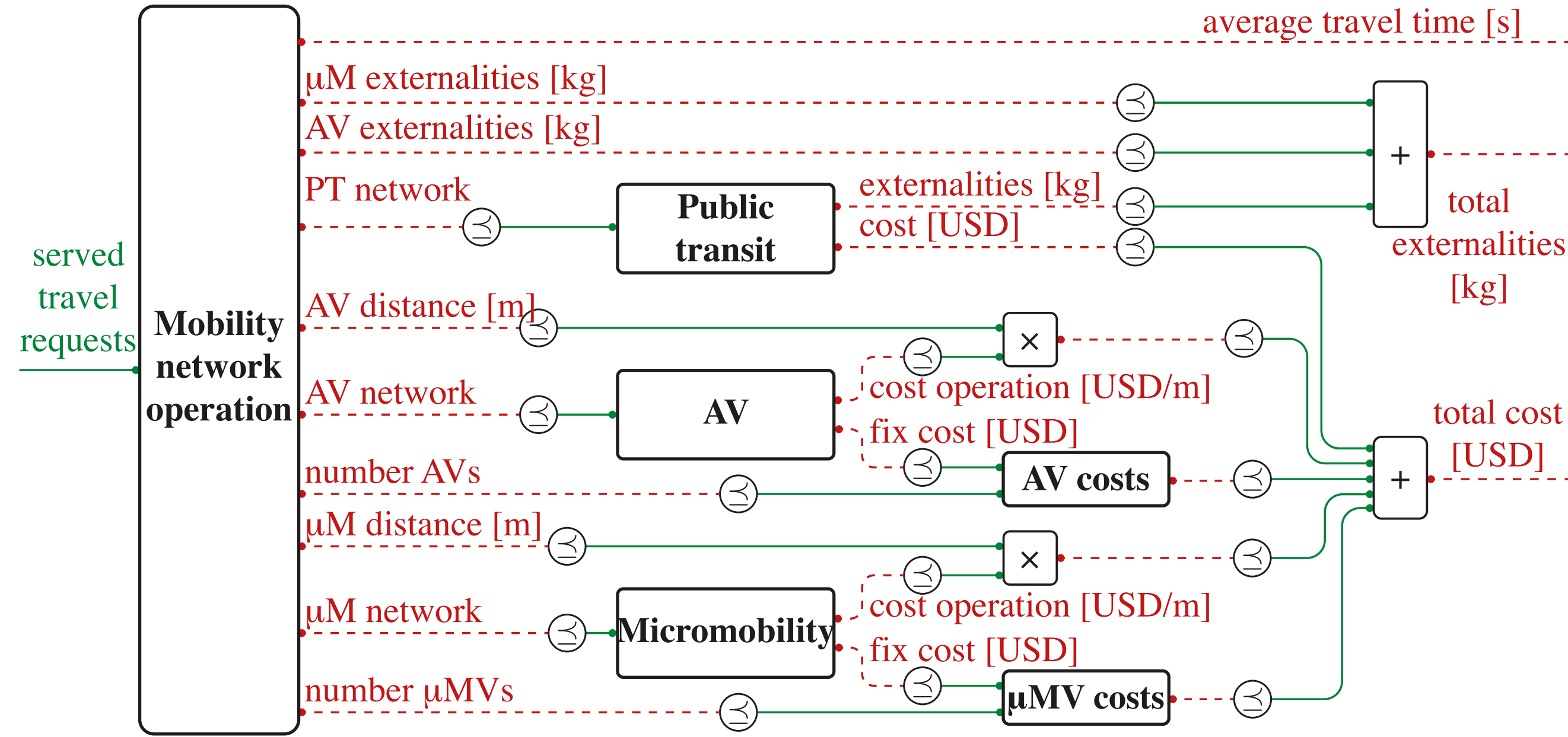
**Interactions** are naturally **hierarchical**:

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[ITSC’21 (*Best Paper Award*), ITSC’23]



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*Next time!*

[ITSC’21 (*Best Paper Award*), ITSC’23]



# Technology is evolving fast

- ▶ Autonomous systems (are?) will be ubiquitous in our lives
- ▶ As engineers, we are getting **closer to the natural sciences**:

*We create things that we do not fully understand, and then we study our creations*



- ▶ In this talk — Two important **challenges** for decision making.
  - ▶ Existing rules are written **by humans for humans**, and require **context** to be **interpreted**
  - ▶ Designed systems need to be **robust to complex, unconstrained environments**, featuring **interactions**



# Behavior requirements for robots are numerous, vague, and conflicting

*Ethics*

*Safety*

*Liability*

*Function*

*Compliance to traffic rules*

*Extensive & diverse  
written by humans for humans*



*Courtesy*

*Culture*

*Example: Boston left*

*Comfort*

*“Does your car have any idea  
why my car pulled it over?”*

PAUL  
NOTH

# Safety for human-driven vehicles

- ▶ Safety (i.e., prevention of *unreasonable risks of driving*) is typically ensured by a mix of:
  - **Certification** of vehicles and drivers
  - **Rules** of the road
  - **Enforcement** by authorities and legal system
  
- ▶ Typically, **rules** rely on fundamental **axioms**, which require **interpretation**

## Fundamental norm in Switzerland:

*All road users must behave in such a way not to pose an obstacle or a danger to other road users*

- ▶ No clear specification of **safety**
- ▶ It is **legal** to break the law to ensure **safety**





# Things that do not work well for AV behavior specification

## ▶ Hard constraints

- What do you do with **infeasibility**?
- Whenever you consider other actors, hard to find **guarantees**

## ▶ Case analysis, finite state machines, ...

- “IF statements kill people”

## ▶ Just relax!

$$J = \alpha J_1 + \beta J_2 + \gamma J_3 + \dots$$

- Hard to re-tune, prone to **overfitting**
- Lack of **transparency**



# What should we do instead?

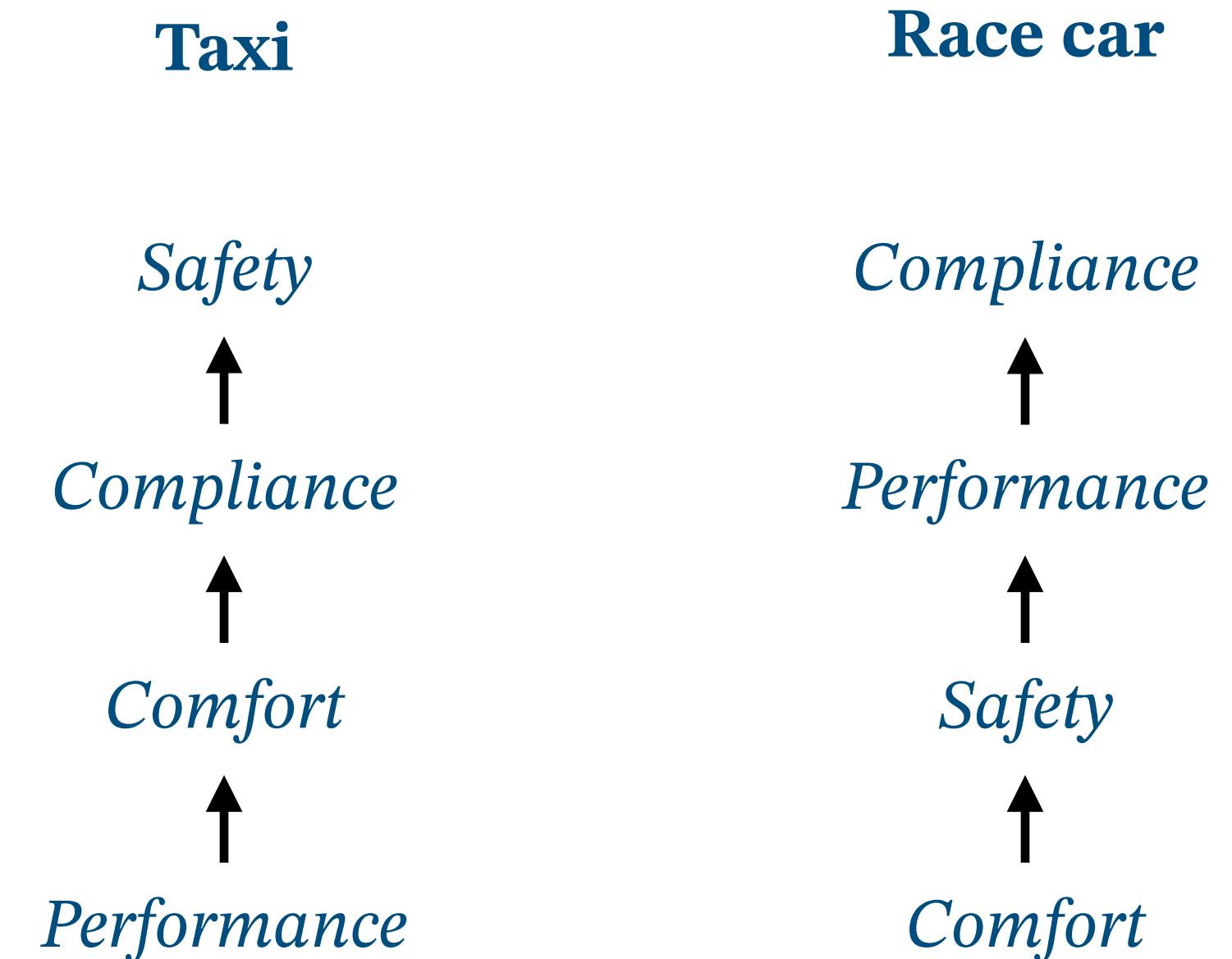


- ▶ Throw the ball at **other stakeholders**
- ▶ Incorporate our **own beliefs** in our algorithms
  
- ▶ Create **transparent** systems
- ▶ Create **customizable** systems
- ▶ **Explain** issues to the public
- ▶ **Engage** with **stakeholders** of the problem (e.g., regulators, liability companies, etc.)



# Minimum violation planning

- ▶ Assume that constraints will be violated, and find the alternative that *least* violates them
- ▶ Define **rules** as a **total order** over realizations
- ▶ **Order rules** according to priority
- ▶ This is practical:
  - Allows **modular definition of behavior**
  - **Easy to predict** what the car will do
  - **Easy to understand** why the car did something
  - One can introduce **tolerances**



What if rules are incomparable, or indifferent?

# We capture the richness of robot behavior requirements via partial orders

- ▶ We can use **pre-orders over rules** to express preferences

“Rule *A* is more important than rule *B*”



“Rule *A* and *B* are not comparable”



“Rule *A* and *B* are indifferent”

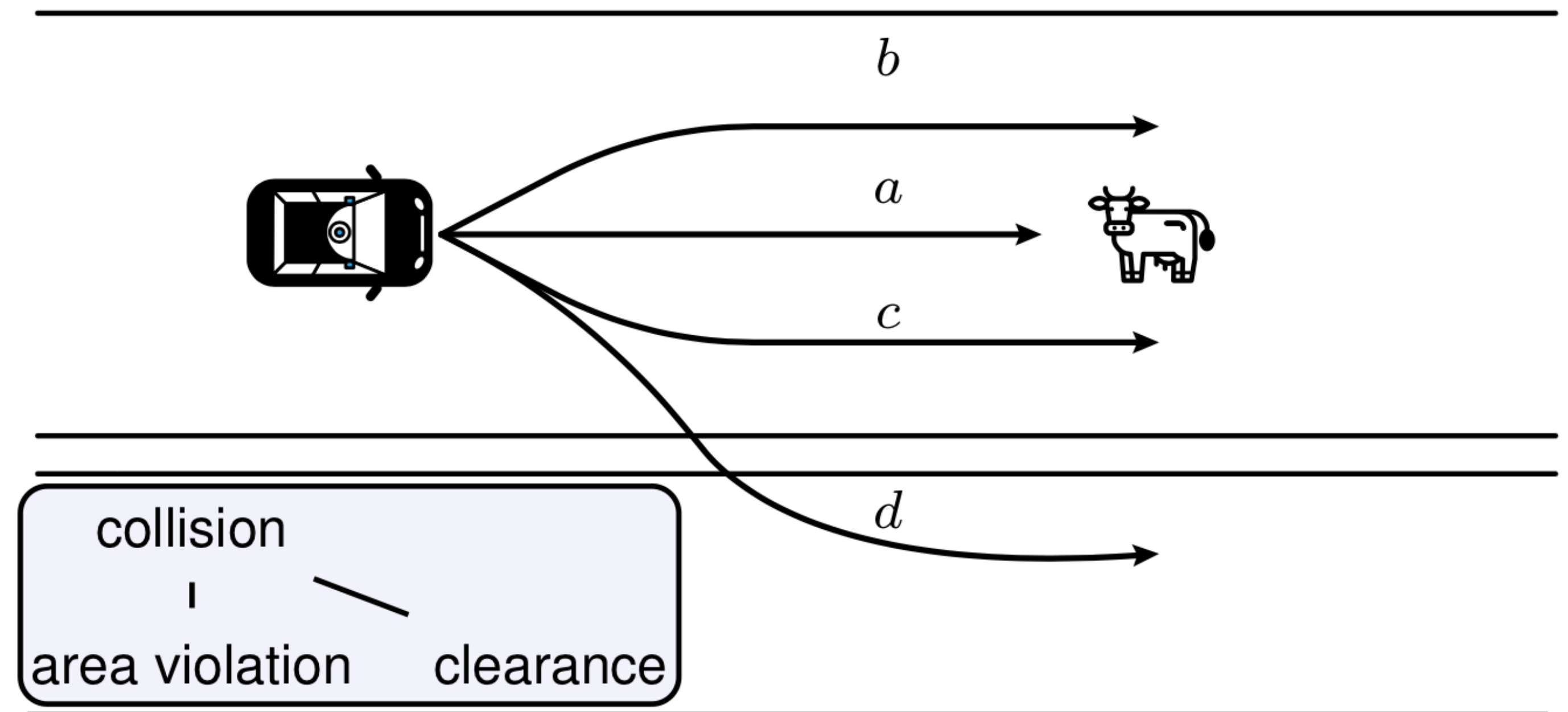


- ▶ Pre-order over rules induces pre-order over outcomes

*b* and *c* are **indifferent**

*b*, *c*, *d* are **preferred** over *a*

*b*, *c* are **incomparable** with *d*





# Minimum violation planning using partial orders, unbridled creativity and good taste

*“The way to get good ideas is to get lots of ideas,  
and throw the bad ones away.” — Linus Pauling*

*creativity*

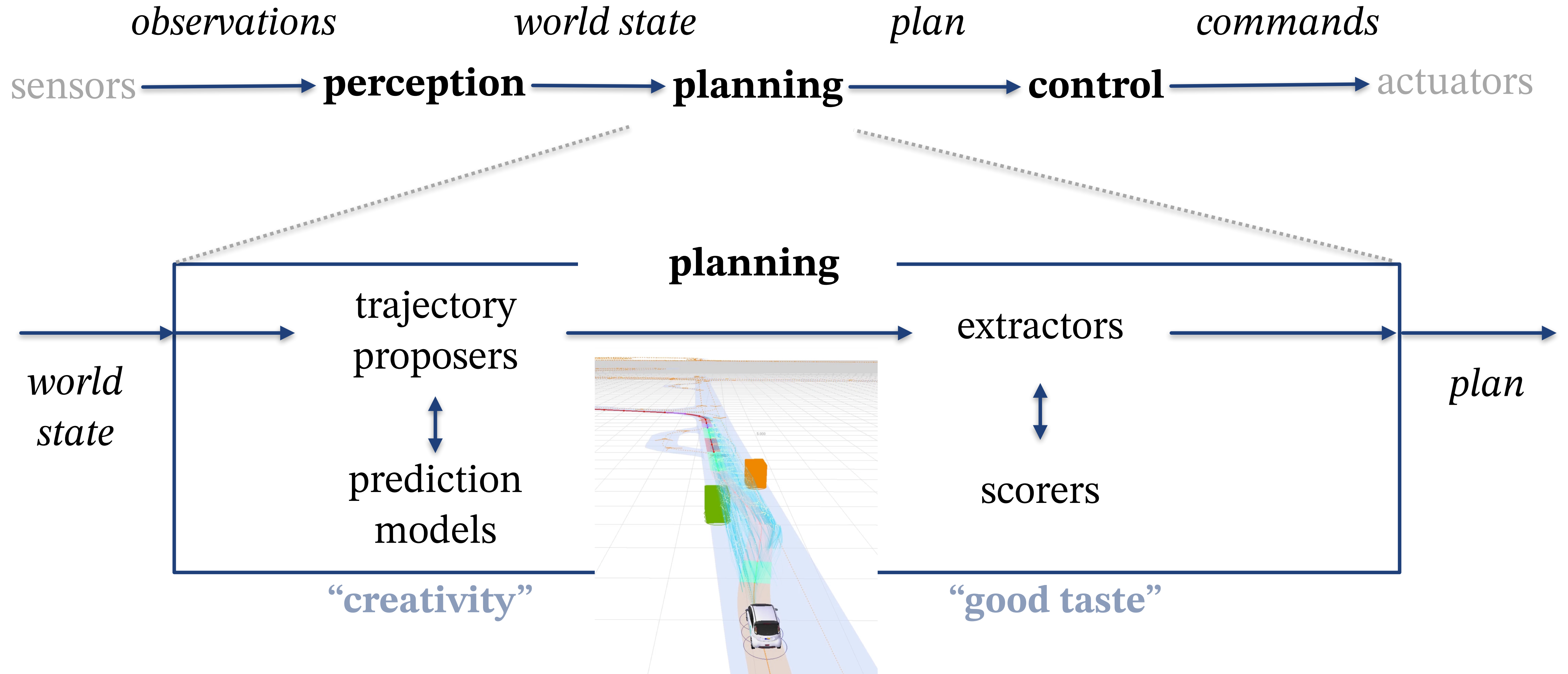


*good taste*





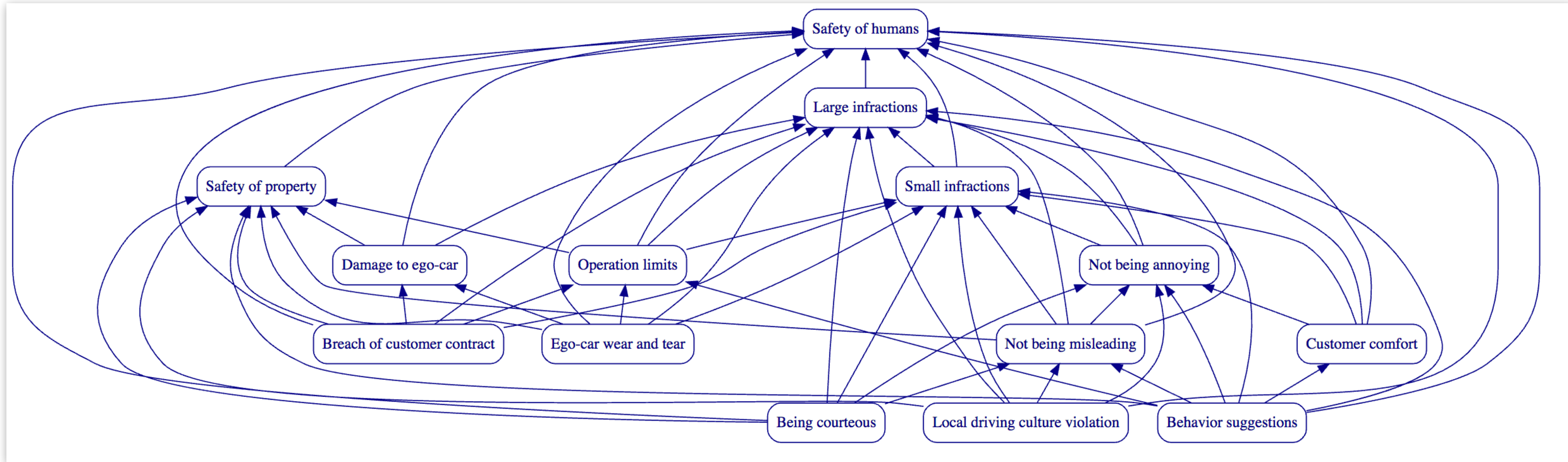
# Minimum violation planning using partial orders, unbridled creativity and good taste





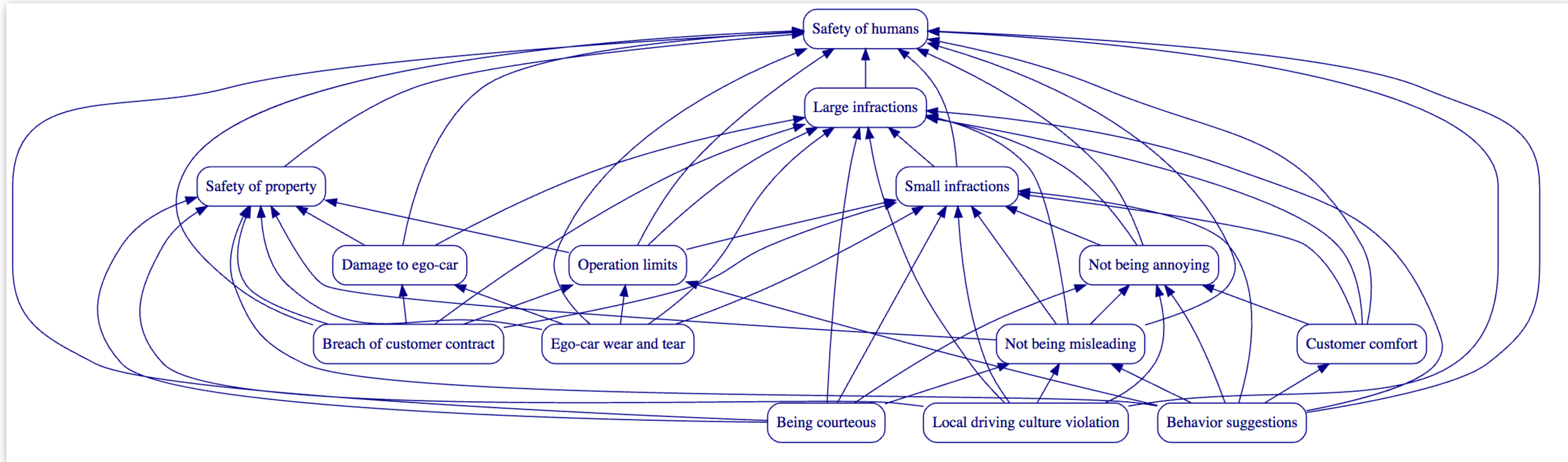
# Defining and ordering rule groups for realistic scenarios

- Estimate: urban driving requires ~200 rules, ~20 rule groups



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All of this is considering ego agents...  
**How do these specifications work with multiple, interacting agents?**



# Posetal Games to deal with highly interactive multi-objective nature of decisions

## ► Games in short:

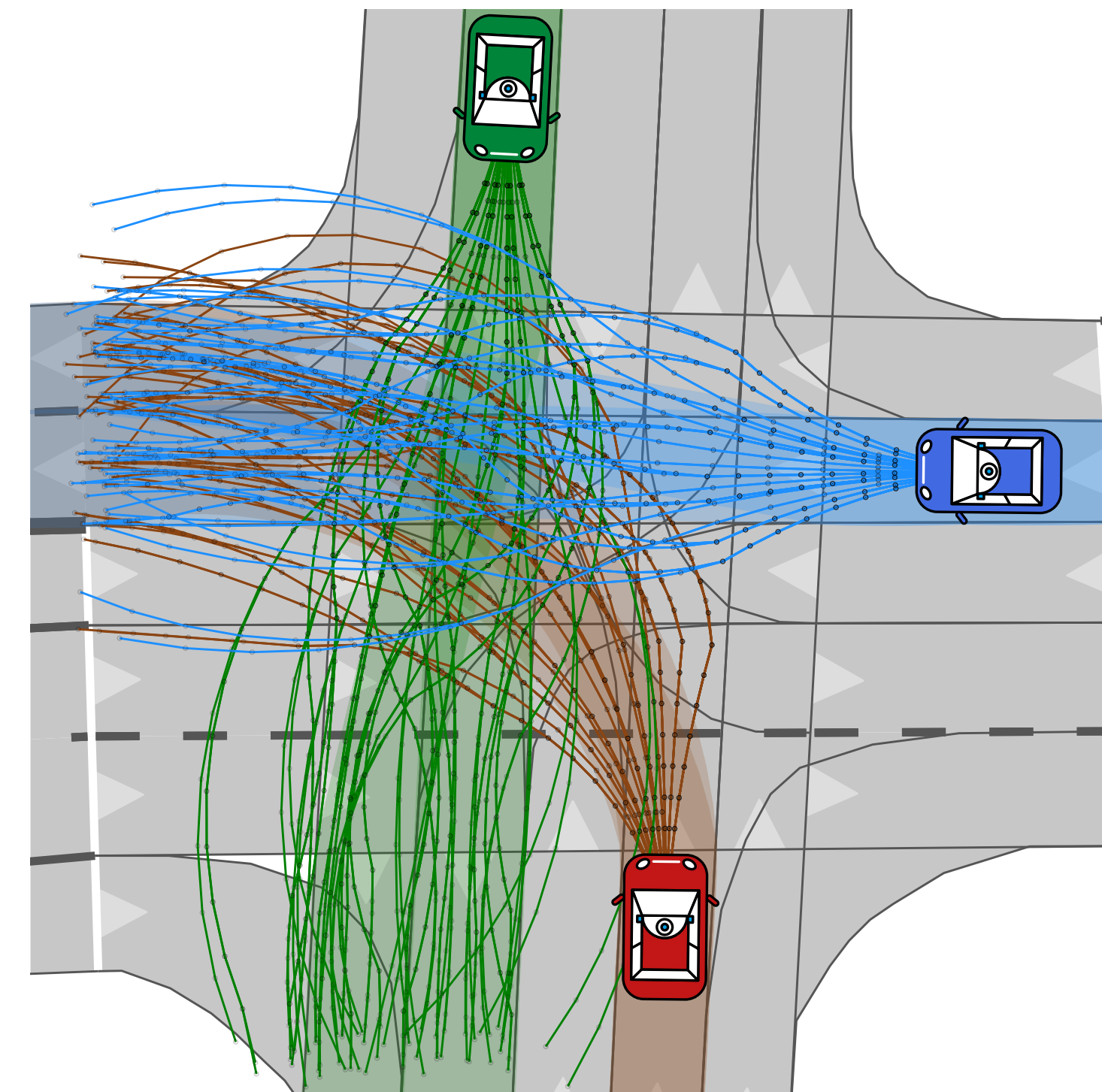
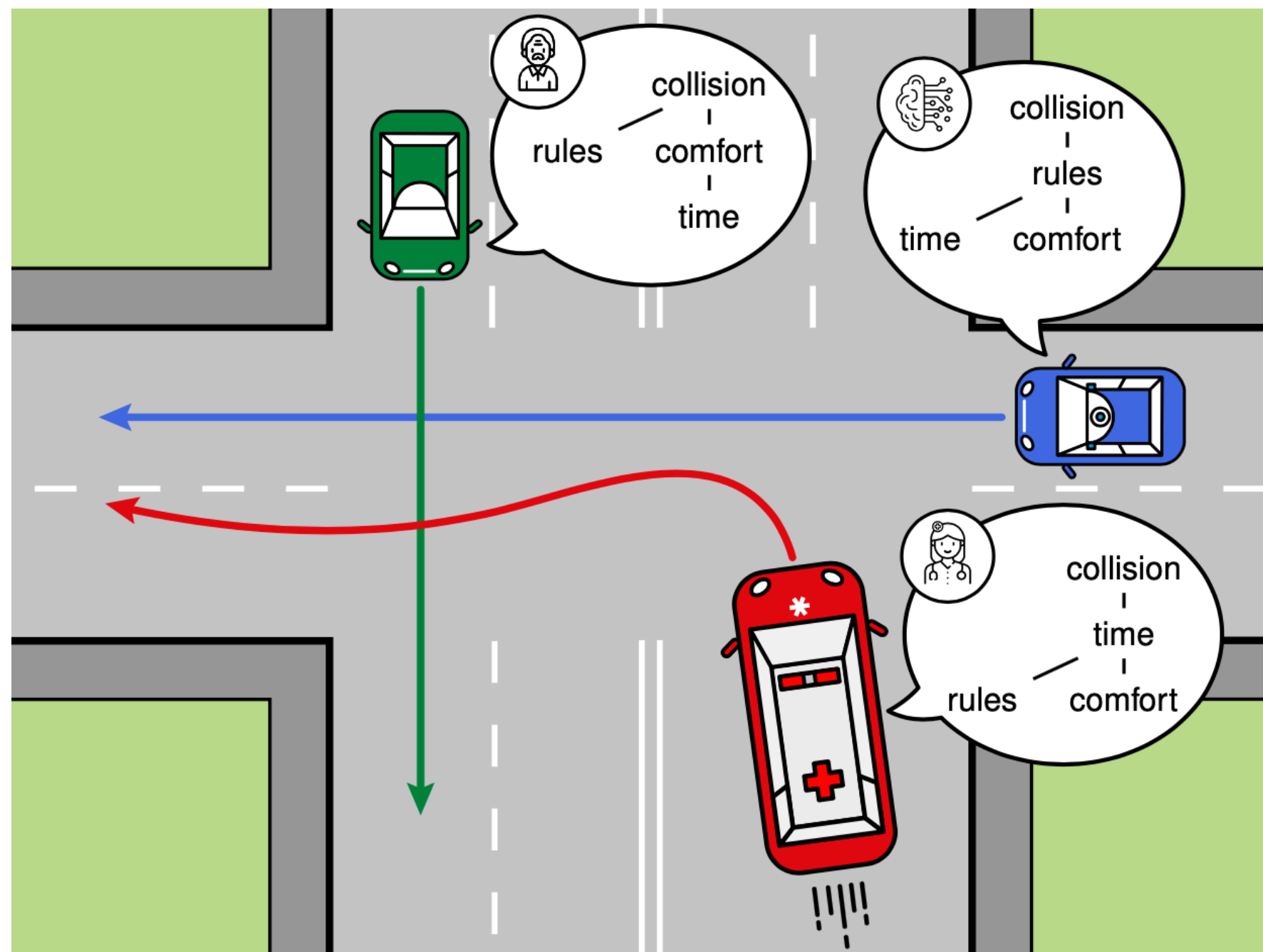
- Each **player** has a **scalar utility function**
- Based on **preferences**, players select an **action** from decision space
- Given joint **action profile** of players, we obtain a **game outcome** for each player via a *deterministic* **metric function**
- **Equilibria** are joint action profiles from which **no player** has **interest to deviate**

# Posetal Games to deal with highly interactive multi-objective nature of decisions

## ► Posetal games in short:

- Each **player** has a **scalar-utility-function** *partially ordered preference* over a set of metrics (scores, costs)
- Based on **preferences**, players select an **action** from decision space
- Given joint **action profile** of players, we obtain a **game outcome** for each player via a *deterministic metric function*
- **Equilibria** are joint action profiles from which **no player** has **interest to deviate**

## *Technical results instantiated in trajectory driving games for urban scenarios*





# Posetal Games to deal with highly interactive multi-objective nature of decisions

- ▶ Posetal games **extend standard notions in game theory**, and
  - Provide **sufficient** conditions for the existence of **Nash equilibria** (via **potential games**)
  - Characterize **efficiency** of **admissible equilibria**
  - Design a **formal, systematic** way to leverage **preference refinement** (e.g., via *estimation*) to **refine equilibria**

