Learning with Certificate Functions for Automotive Systems

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Certificate functions from control theory prove desired system properties.

**Lyapunov Function**
- Certifies stability of a fixed point

**Barrier Function**
- Certifies invariance of a region

**Contraction Metric**
- Certifies ability to track arbitrary trajectories
Neural control certificates can help with

Single-agent certified learning-based control from high-dimensional inputs (robust, online, adaptive)

Generalization in a large fleet of agents

Provide insights on other uncontrolled road participants reacting to the autonomous cars

Combined with RL for certificate-carrying RL

[Dawson RAL'22, Dawson CoRL'22, Dawson TR-O’23, Garg CSS-L’23, Tong [CRA’23]

[Qin ICLR’21, Qin RAL’22, Zhang CoRL’23]

[Meng IROS’21]

[So RSS’23]
Neural certificate control

To solve LiDAR-based navigation, from a control certificate point of view, a robust Control Lyapunov Barrier function (rCLBF) can serve the purpose of certifying safe reach-avoid problems.

\[
\begin{align*}
V(x_{\text{goal}}) &= 0 \\
V(x) &= 0, \forall x \in X \setminus x_{\text{goal}} \\
V(x) &> 0, \forall x \in X_{\text{safe}} \\
V(x) &< c, \forall x \in X_{\text{unsafe}} \\
\inf_{u} L_{f_{\theta}} V + L_{g_{\theta}} Vu + \lambda V(x) &\leq 0, \forall x \in X \setminus x_{\text{goal}}
\end{align*}
\]

**Theorem:** If we can find such a \( V \) for a control policy \( u \), then the closed-loop system is robustly safe and stable in terms of goal-reaching.

There are many approaches to find such a \( V \), such as SoS, Simulation-guided synthesis. But the computational complexity has been a bottleneck so far.
Neural certificate control

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V(x) > 0, \forall x \in \mathcal{X} \setminus x_{\text{goal}} \\
V(x) \leq c, \forall x \in \mathcal{X}_{\text{safe}} \\
V(x) > c, \forall x \in \mathcal{X}_{\text{unsafe}}
\]

\[
\inf_{\mathcal{U}} \sup_{\mathcal{X}} \left( V^2(x_{\text{goal}}) + a_1 \frac{1}{N_{\text{safe}}} \sum_{x \in \mathcal{X}_{\text{safe}}} \max(V(x) - c, 0) + a_2 \frac{1}{N_{\text{unsafe}}} \sum_{x \in \mathcal{X}_{\text{safe}}} \max(V(x) - c, 0) \\
+ a_3 \frac{1}{N_{\text{train}}} \sum_{x \in \mathcal{X}} r(x) \sum_{i} \max(L_{f\theta_i} V + L_{g\theta_i} Vu + \lambda V(x), 0) \right)
\]

[Dawson 21] \( u \) and \( V \) can be represented as NNs, with loss \( \mathcal{L} \) being
Neural certificate control

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$$
Neural certificate control

Permutation-invariant embedding of lidar measurements

Observation-based CBF

Observation-based CLF
Neural certificate control can achieve 100 Hz (real-time) control, versus 4 Hz for robust MPC.

Our approach (100 Hz)  
Robust MPC (10 Hz, less than real-time)  
Robust MPC (4 Hz, real-time)
To handle multi-agent systems and generalize well to different settings, we need to fully decentralize the controller and neural certificates.

Songyuan Zhang, Kunal Garg, and Chuchu Fan, “Distributed Safe Multi-agent Control Using Neural Graph Control Barrier Functions.”
Graph control barrier functions (GCBF)

A multi-agent system is naturally a graph \( \mathcal{G} = (V, E) \)

- \( V = V_a \cup V_o \), where \( V_a \) are agents and \( V_o \) are the hitting points of LiDAR rays in the observation
- The edges \( E \) are defined between each of the observed points and the observing agent when the distance between them is within the sensing radius \( R \).

A GCBF for the graph is defined as

\[
\begin{align*}
    h(e_i, v_i) &> 0 \text{ for } x_i \in S_i, \text{safe} & \quad & e_i: \text{edge features} \\
    h(e_i, v_i) &< 0 \text{ for } x_i \notin S_i, \text{safe} & \quad & v_i: \text{node features} \\
    \sup_{u_i} \left[ \dot{h}(e_i, v_i) + \alpha(h(e_i, v_i)) \right] &\geq 0 \text{ for } x_i \in S_i, \text{safe} & \quad & \alpha: \text{class-}\mathcal{K} \text{ function}
\end{align*}
\]

Theorem: Existence of a GCBF guarantees the safety of the multi-agent system from non-colliding initial conditions.
GCBF learned as Graph Neural Networks (GNN)

The GCBF is trained without explicit models of the obstacles, but some randomly sampled point in the point clouds.

- Other agents
- End points of LiDAR
Graph control barrier functions (GCBF)

Trained with 16 Dubin cars, the resulting controller can be deployed on an arbitrary number of agents with the same dynamics.
Graph control barrier functions (GCBF)

Trained with 16 Dubin cars, the resulting controller can be deployed on an arbitrary number of agents with the same dynamics, even with increased density.
GNN-based neural CBF-based control

Increase density (similar travel duration)

Keep Density (travel longer)

Keep distance (similar travel duration)

Number of agents

Success rate

SimpleCar

SimpleDrone

DubinsCar

GCBF

MDCBF (Our previous version)

MAPPO
Graph control barrier functions (GCBF)

With LiDAR inputs, the decentralized controller can handle unseen (uncontrolled) obstacles of varying sizes and speed.
GNN-based neural CBF-based control

![Graphs showing reaching rate and success rate with varying number of obstacles.](image-url)
Neural certificates can help build reactive other road participants

Human intents (e.g., collision avoidance, traffic rules) can be learned as neural certificates from real-world traffic and be enforced in the learned human behavior models [Meng 21].
Inserting certificates in learned behavior models

It turns out the human behavior model learned in this way can generalize to unseen scenarios more realistically. [Meng 21]

(a) Fixed trajectories  (b) Imitation Learning  (c) Ours

A (never seen before) vehicle attempts to drive through a crowd with different pedestrian models.
We can also insert traffic rules such as safety, stop signs, and traffic lights as neural certificates to each learned model. Deployed on a more complex traffic scene, our agents always follow the traffic rules.
Neural certificates for automotive systems

Neural certificates provide a quantifiable way to certify learning-based control from sensor inputs.

Controllers from neural certificates can be designed to be robust, online, and adaptive.

Neural certificates can provide fully decentralized, scalable, and generalizable collision-avoidance control for vehicles.

Neural certificates can capture some human intents when building models for other road participants.

And many more applications using neural certificates!